An experimental method to measure the ground state

HyperFine Splitting in Muonic Hydrogen

PROTON FINIE SIZE

PROTON POLARIZABILITY

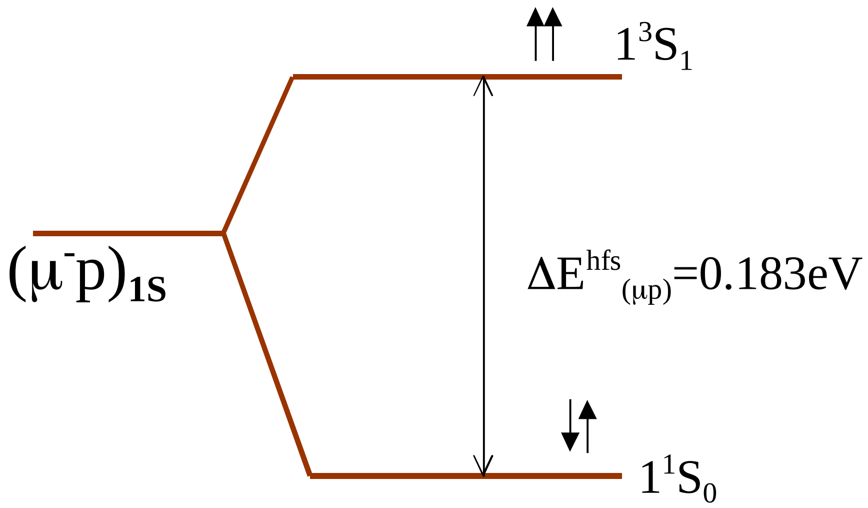
DYNAMIC FORM FACTOR

EFFECTIVE RADIUS

WAVEFUNCTION PATTERN

Andrea Vacchi INFN Trieste

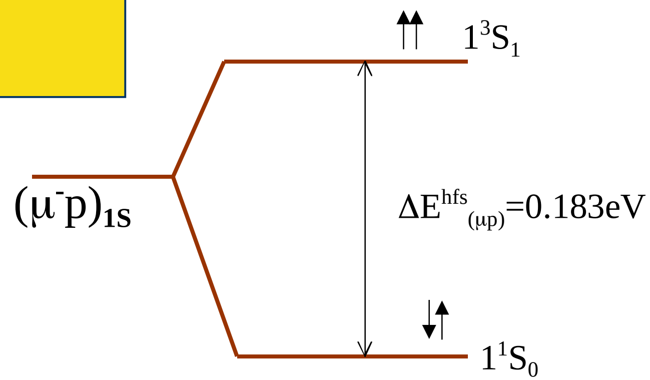
M1 transition in μP



- **A precision of 10 ppm on $\Delta E^{hfs}_{\mu p}$ can be reached with a pulsed tunable infrared laser.**
- If the achieved exp. precision is $\Delta E^{hfs}_{\mu p} = 0.183 \pm 2.5 \times 10^{-4}$ eV, since δ^{POL} is about 0.46×10^{-3} we get to δ^{POL} ±20% and this would be the first measurement and not a simple limit.
- $\lambda = 6.62 \times 10^{-4}$ **cm, corresponds to the region of the infrared**

Introduction

•Laser-stimulated singlet to triplet transition of muonic hydrogen **μp**



- A measure of the hyperfine splitting in the muonic hydrogen μp ground state level.
  - The experimental value of the $^3S_1$ - $^1S_0$ energy difference if measured with a relative accuracy of 10^{-4} or better leads to information on the magnetic structure and the *polarizability of the proton*
  - *The μ hydrogen experiment delivers complementary information to experiments on hydrogen,* (momentum transfer dependence).

HYDROGEN HYPERFINE SPLITTING

- The hyperfine splitting $\Delta E^{hfs}_{\exp}$ of the hydrogen ground state level has been measured with a very high accuracy
  - $\Delta E^{hfs}_{\exp}$ =1420.40575176679(9) MHz this result, that exceeds the accuracy of the theoretical value, is the most precise experimental result ever obtained.
  - In Muonic hydrogen the light electron is replaced by the heavy muon. In the μp system the relative contribution of the various effects change because of the m_{μ}/m_{e} ratio, the momentum transfer is of the order of the mass of the orbiting particles.
  - $\Delta E^{F}(\mu p)$ = Fermi term for muonic hydrogen is

$$\Delta E^{F} = 44.114 \text{ THz}$$

Hydrogen hyperfine splitting

$$\Delta E_{th}^{hfs} = \Delta E^{F} \times (1 + \delta^{QED} + \delta^{FF} + \delta^{POL})$$

$$\Delta E^{F} = \frac{8}{3} \times \alpha^{4} c^{2} \frac{\mu_{p} M_{p}^{2} M_{l}^{2}}{(M_{p} + M_{l})^{3}}$$

- ΔE^F = Fermi term, leading order effect
- $M_{p,l}$ = masses of proton and electron
- μp = proton magnetic moment, in nuclear magnetons.
- δ^{QED} = correction term allowing for relativistic and radiative effects
- δ^{FF}, δ^{POL} correction terms taking into account the structure of the proton
- In Hydrogen ΔE^{hfs} is dominated by the QED contribution, in Muonic Hydrogen the nuclear structure correction terms δ^{FF}, δ^{POL} tends to dominate
- $\Delta E^F(\mu p)$ = Fermi term, for muonic hydrogen is ΔE^F = 44.114 THz*

δ^{QED} first order does not depend on m_l/m_p

- δ^{QED} , the contribution from higher-order quantum-electro dynamical effects to the hfs, allows for relativistic and radiative effects. The leading terms in δ^{QED} does not depend on m_l.
  - Electron anomalous magnetic moment of order $O(\alpha)$ and the Breit corrections of order $O(\alpha^2)$, this are the dominating terms and are independent on m_l.
  - Theoretical results for contributions to δ^{QED} up to the order $O(\alpha^3)$ are known, the remaining uncertainty due to δ^{QED} on ΔE^{hfs}_{th} accounts therefore for less than 10^{-6}, (0.12ppm)*.

δ^{FF}, δ^{POL}

- δ^{FF}, δ^{POL} describe the effects of the internal structure of the proton*;
  - δ^{FF}, δ^{POL} vanish in the theoretical expression for the hyperfine splitting of the point like nuclei μ^+ and e^+ muonium and positronium

δ^{FF}

– δ^{FF} is associated to the structure of the proton, it accounts for the static effects corresponding to the approximate picture of a rigid spatial distributions of the proton nuclear charge and magnetic moment distribution. Associated to the proton electromagnetic form factors as extracted for e-p elastic scattering data, δ^{FF} is expressed* in terms of some integrals of the proton form factors, in the lowest order approximation

$$\delta^{FF} = -2\left(\frac{\alpha \times c}{\hbar} \times \frac{m_l \times m_p}{(m_l + m_p)}\right) \times \langle R_{pr} \rangle \qquad \delta_e^{FF} \approx 4.5 \times 10^{-5}$$

$<R_{pr}>$ being a combination of the charge and magnetic radii of the proton. The uncertainty on δ^{FF} of about 2% is due to the poor knowledge of the proton e.m. form factors for higher momentum transfer and adds for about 1 ppm to the uncertainty of ΔE^{hfs}_{th}

δ^{POL}

- δ^{POL} incorporates the corrections to the hyperfine splitting due to the fact that the proton charge and the magnetic moment distribution are not absolutely rigid but are rather polarized by the orbiting electron.
- A model independent expression for δ^{POL} in term of experimentally observable quantities do not exists.
- The most reliable result about the magnitude of δ^{POL} is an upper bound; in order to make explicit the proportionality of δ^{POL} to m_l we can represent it in the form

$$\left|\delta^{pol}\right| \leq C^{POL} \times \left({m_l}/{m_{el}} \right), \ldots\ldots\ldots C^{POL} \approx 4.10^{-6}$$

From the above equations we can easily obtain estimates for the same corrections to the hyperfine splitting of the muonic hydrogen ground state by simply rescaling the corresponding values for the hydrogen with the appropriate power of (m_l/m_e)

Rescaling with (m_l/m_e)

	Hydrogen			Rescaling factors	Muonic Hydrogen		
	Order of magnitude	Relative error	Relative uncertainty caused on ΔE^{hfs}		Order of magnitude	Relative error	Relative uncertainty caused on ΔE^{hfs}
ΔE^F	1.42GHz	10^{-7}	10^{-7}	$(m_l/m_e)^2$	56.8THz	10^{-7}	10^{-7}
δ^{QED}	$O(\alpha)$	$<10^{-6}$	$<10^{-6}$	1	$O(\alpha)$	$<10^{-6}$	$<10^{-6}$
δ^{FF}	10^{-4}	2%	10^{-5}	(m_l/m_e)	10^{-2}	≈2%	10^{-3}
δ^{POL}	$<4.\ 10^{-6}$	100%	$4.\ 10^{-6}$	(m_l/m_e)	$>10^{-3}$	100%	10^{-3}
$\delta^{FF}+\delta^{POL}$	10^{-4}	10%			10^{-2}	10%	
ΔE_h^{hfs}	10^{-8}				861 MHz	10%	$5.5\text{x}10^{-5}$

The QED correction term will conserve his order of magnitude $O(\alpha)$ and uncertainty.

δ^{FF}, δ^{POL} , gain a factor of m_l/m_e, about 10^2

The upper bound to the proton form factor δ^{FF}, now amounts to $0.6\text{x}10^{-2}$

The proton polarizability δ^{POL} , rescales to $0.8\text{x}10^{-3}$

The relative contribution of the hadronic vacuum polarization to the hfs splitting of the ground state in muonic hydrogen is as large as 5.5 x 10^{-5} , it is due to the relative momentum of the particles which reaches values of the order of the hadronic masses. This shall be taken into account for a precise comparison to experimental results on δ^{FF}, δ^{POL} .

Hydrogen and Muonic hydrogen

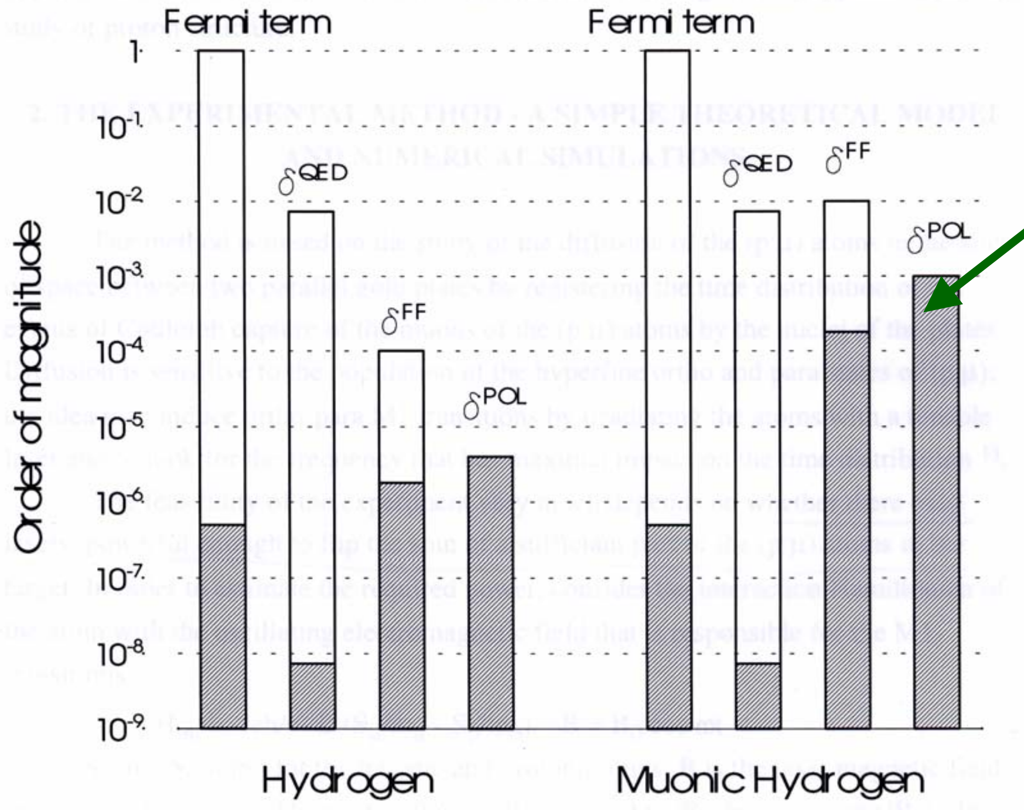


Fig.1. Contributions to the hyperfine splitting of the ground state energy levels of hydrogen and muonic hydrogen atoms from the Fermi term ΔE^{Fermi}, the QED correction δ^{QED}, proton finite size correction δ^{FF} and proton polarizability correction δ^{POL}. The shadowed areas correspond to the uncertainty of their values.

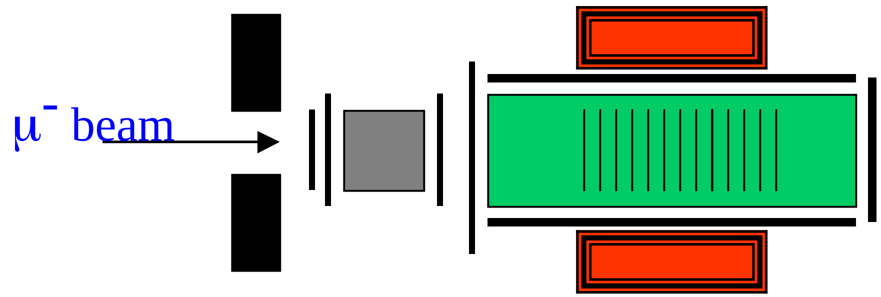
The fantastic precision of the experimental value of ΔE^{hfs} for hydrogen makes it sensitive to QED corrections, proton finite size and proton polarizability. δ^{FF}, δ^{POL} can not be extracted from the single experimental value. A measurement of the hyperfine splitting in muonic hydrogen with an accuracy of 10^{-4}-10^{-6} will provide independent data on the combination δ^{FF}, δ^{POL} for which only upper bounds exists up to now*.

Summary

- High precision measurements of ΔE^{hfs} in the (e^- p) and (μ^-p) systems are complementary to one another.
- The QED correction term will conserve his order of magnitude O(α) and uncertainty.
- δ^{FF}, δ^{POL} , gain a factor of m_l/m_e, about 10^2.
- In (μ^-p) the upper bound to the proton form factor δ^{FF}, is rescaled to $0.6\mathrm{x}10^{-2}$,
- the proton polarizability δ^{POL} now amounts to $0.8\mathrm{x}10^{-3}$.
- The relative error of δ^{FF}, because of the larger contributions to the e.m. form factor, from the worse known higher momentum transfer will be larger than 2.5% in comparison to normal hydrogen.
- The overall uncertainty on δ^{FF}, does not exceed 2 $\mathrm{x}10^{-4}$ and a measurement of ΔE^{hfs}_{exp} with an accuracy of 10^{-4} will establish a relation between δ^{FF} and δ^{POL} that holds with the same accuracy. Improving the knowledge of the e. m. form factor of the proton so to restrict the error of δ^{FF} to 1% would *reduce the uncertainty on* δ^{POL} *by a whole order of magnitude*

- When a μ^- stops in a H_2 target, a neutral atom (μ^-p) is formed with an initial energy > 1 eV and in a statistical mixture* of the two hyperfine states F=0 and F=1.
- At 10 atm the termalization is fast, the $(\mu^-p)_{1S}$ will soon (100-200ns), be present in the singlet F=0 state with an average kinetic energy near to the thermal one

The experimental method, requires an intense low energy pulsed muon Beam and an high power tunable pulsed laser



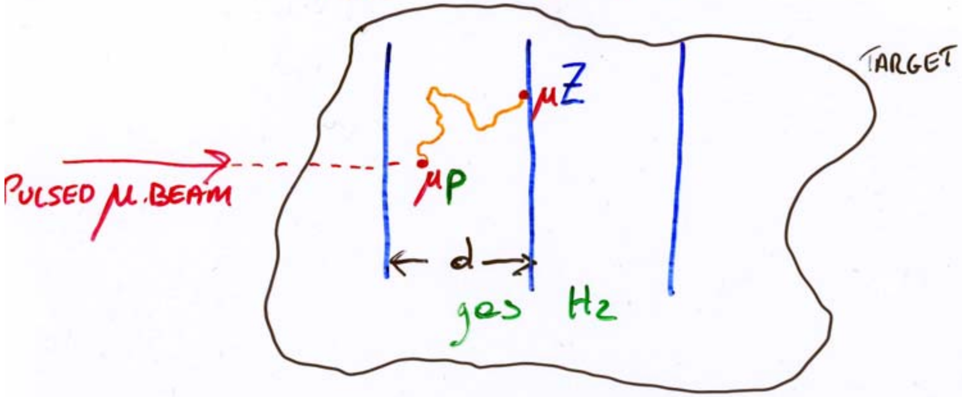
10 Atm. H2

Hydrogen gas target

X-ray Detectors

Beam telescope
and anti-coincidence system

THE DIFFUSION



FEW NANO SECONDS AFTER HIS FORMATION IN A GAS H_2 TARGET THE μp HAS LOST THE INITIAL SPEED AND REACHED THE

SINGLET SPIN STATE

$$\mu p + p \rightarrow \mu p + p \;; \qquad \sigma_S \simeq 1 \times 10^{-20}\,cm^2$$

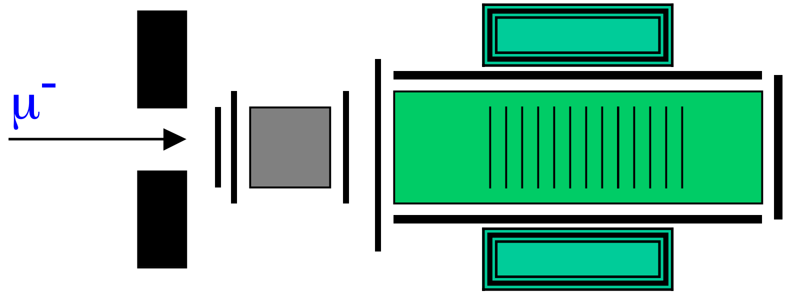
PL 78B 1978 355

DIFFUSING WITH THERMAL SPEED THE μp EVENTUALLY REACHES THE WALLS (THIN FOILS) WHERE THE FAST TRANSFER REACTION OCCOURS:

$$\mu p + Z \rightarrow (\mu Z)^* + p$$
$$\quad \hookrightarrow \mu Z + X_c$$

Experimental method

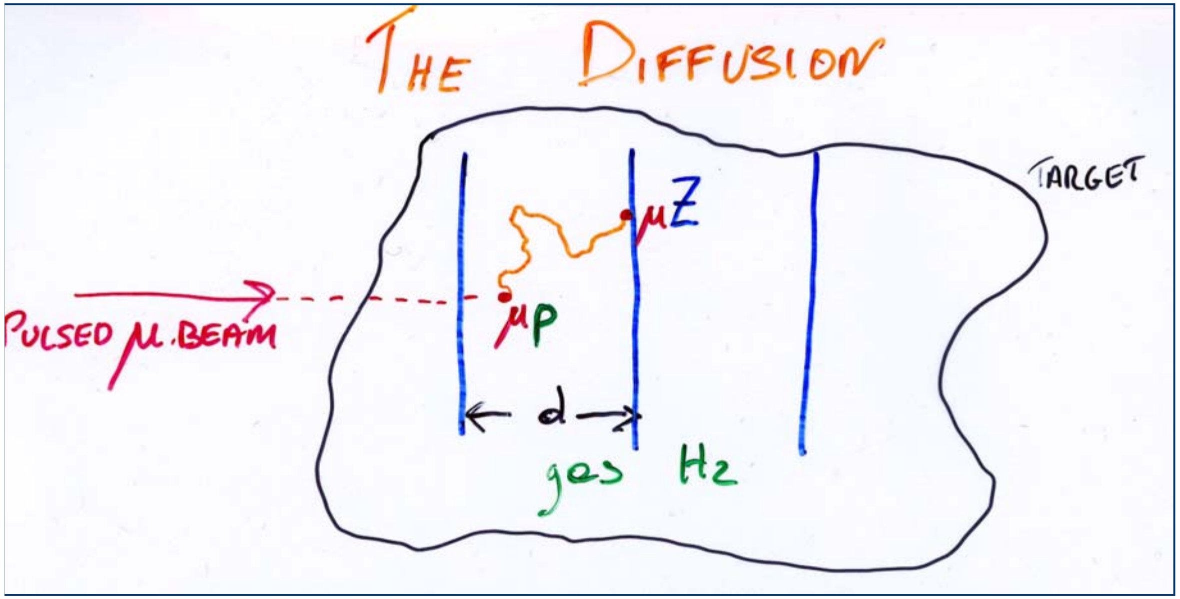
$$\mu^- p + Au \rightarrow \left(\mu^- Au\right)^* + p$$

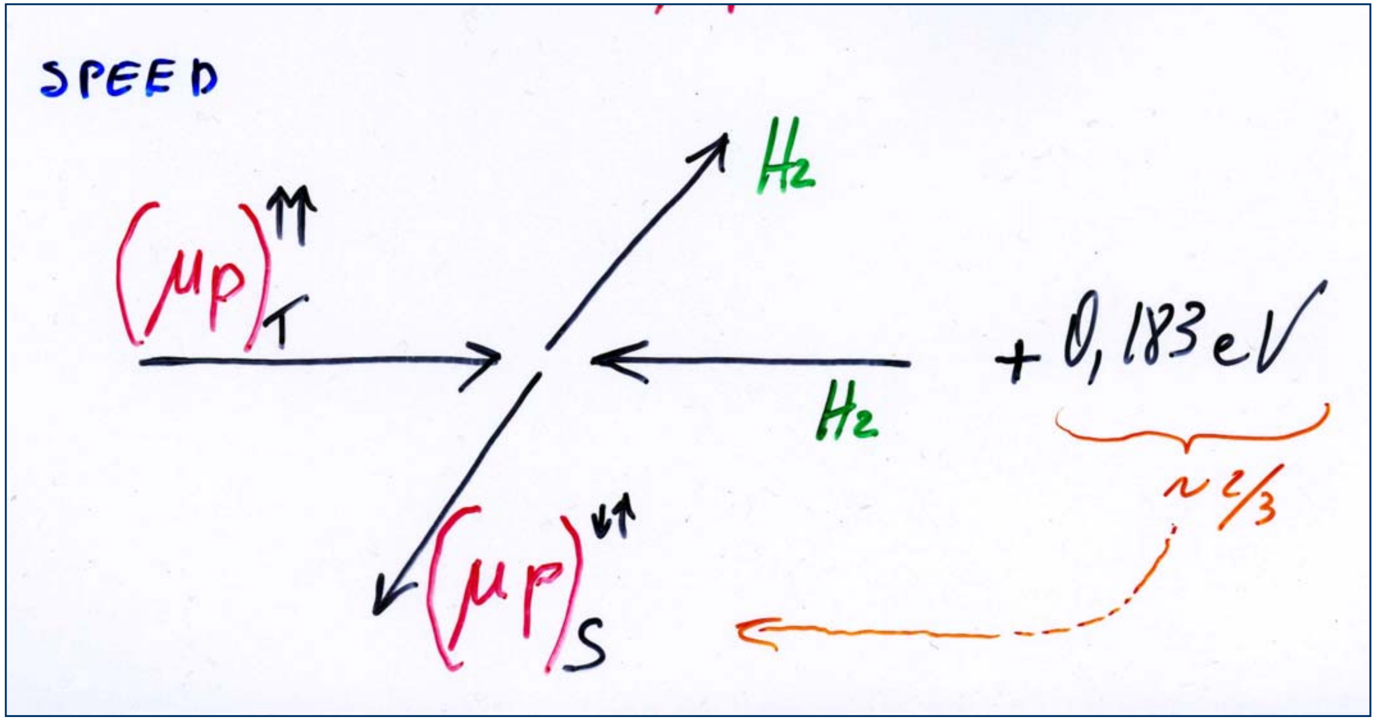


- Thin foils of gold are placed in the target, some of the $(\mu^- p)_{1S}$,reaching the gold surface, transfers the μ^- to form an excited muonic-gold ion.
- Prompt characteristic deexcitation x-rays (5.8 MeV) will be emitted from the $(\mu^- Au)^*$ when the $(\mu^- p)$ atoms reach the gold foils.
- The time distribution dNx/dt of this x-rays can be observed. This distribution will be mainly function of the scattering cross section $\sigma_{0\text{->}0}$ of the muonic system against H_2 molecules, the distance d between the gold plates, the pressure p and the temperature T of the gas <u>and the velocities of the colliding systems.</u>

Spin flip transition

- Supposing to induce by laser light a the time t_{laser} the spin flip transition $\uparrow\downarrow \longrightarrow \uparrow\uparrow$ since $\sigma_{T\text{-}S} \sim 20\ \sigma_S$, very fast the μp atom reaches the singlet state $\uparrow\downarrow$ again BUT conserving a fraction of $\Delta E^{hfs}_{\mu p}$ and moving at an higher speed.
- The effect of this laser induced transition must be visible in the distribution of X-rays from the transfer reaction dNx/dt. More muons have at a given time a sufficient speed to escape the decay channel and reach the foils.
- Inducing the singlet to triplet transition by irradiating the atoms with a tunable laser and looking for the frequency that has the maximal impact on the time distribution.


THE DIFFUSION
TARGET
μZ
μp
PULSED μ BEAM
d
gas H2


SPEED
H2
(μp)T ↑↑
+ 0,183 eV
H2
~2/3
(μp)S ↓↑

The idea is to induce the ortho-para M1transiton by irradiating the atoms with a tunable laser and and look at the frequency that has the maximal impact on the time distribution.

A Monte Carlo Calculation shows the effect.
The (μ^-p) system is made to disappear either for μ^- decay or because it reaches the gold surface

P = 10 atm

T = 300K

D = 1mm

$P(\lambda_r)$ = 1% at 44THz

300000 (μ^-p)

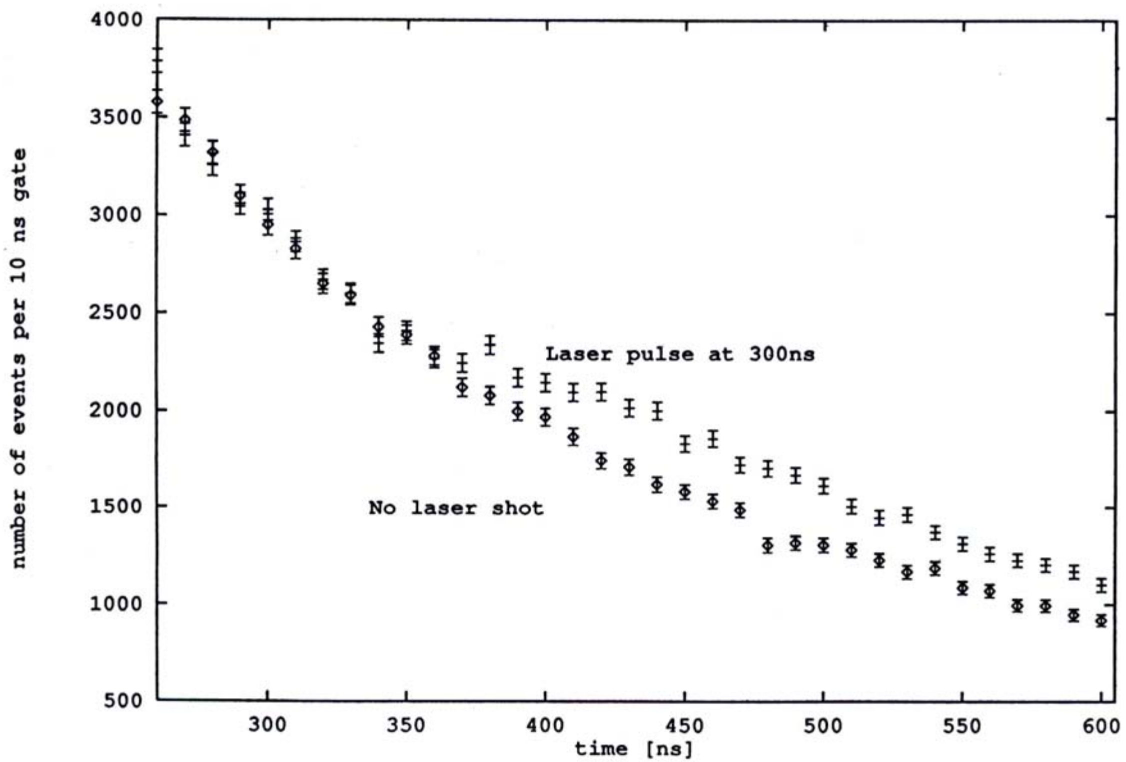


Fig. 2. Simulated time distribution dN_X/dt of the events per 10 ns gate for pressure $P=10$ atm, temperature $T=300$ K, distance between the gold foils $d=1$ mm and transition probability 1%. 300000 $(\mu^-p)_{1S}$ atoms are present in the target at the initial moment. The upper curve represents the time distribution of the events when a laser pulse of width 20 ns is injected at time $t_\ell=300$ ns; the lower curve represents the time distribution obtained when no laser pulse is injected.

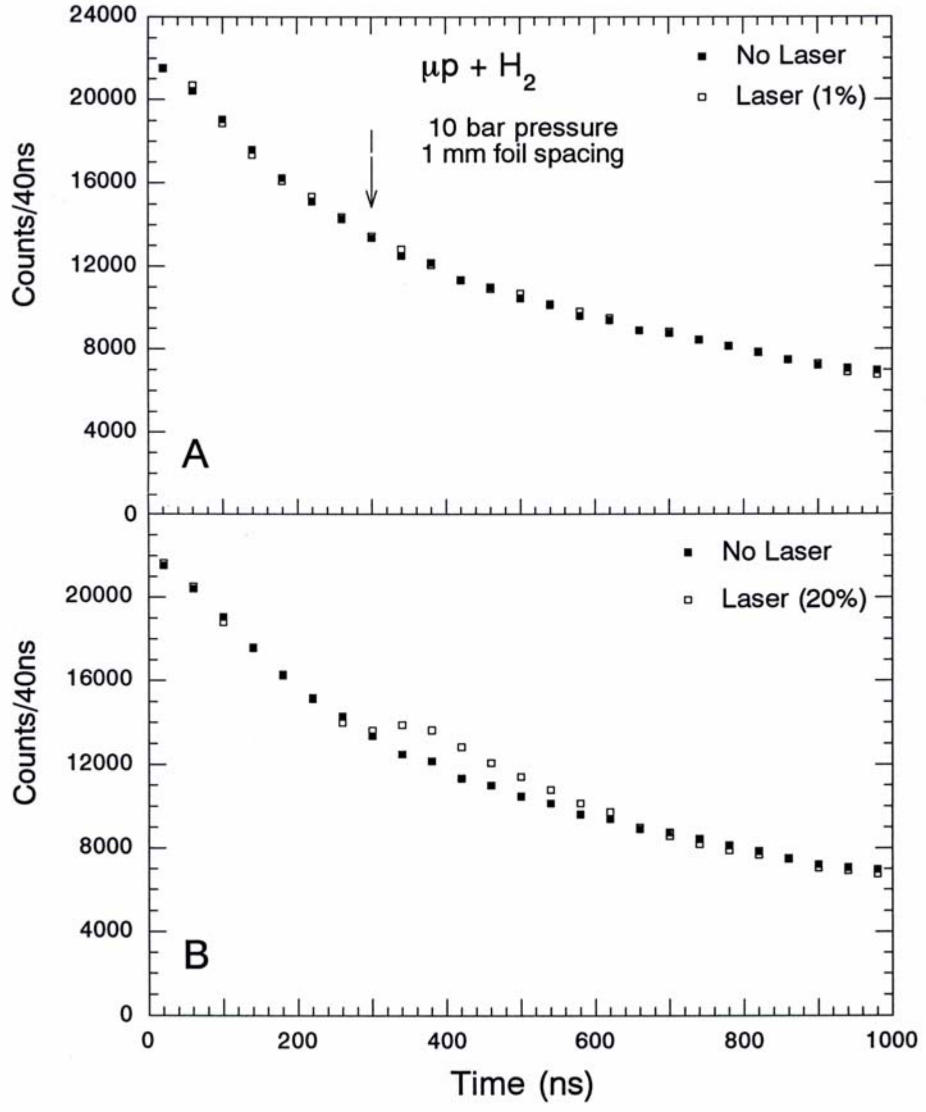
Simulating

- The lower curve is in qualitative agreement with previous observations*.
- Once the $(\mu^- p)_{1S}$ system is thermalized a laser pulse, few tens of ns width, of a proper wavelength λ is sent in to the target in order to induce singlet to triplet transitions.
- When such transition occurs, due to a "jump mechanism * " in a very short time the F= 1 state will be converted, due to collisions with H_2 molecules, to the F=0 singlet state ($\sigma_{1\to 0} \sim 10^{-18}$ cm^2)**
- the singlet state retains about 0.12 eV of kinetic energy. This is much higher than the thermal kinetic energy possessed before the laser shot. Therefore after the laser shot if the transition has occurred the diffusion process will be altered and a change in the dN/dt distribution will occur.

notes

- Different authors* have simulated the Experiment using a different Monte Carlo program,
- The program uses differential molecular scattering cross sections ($\mu^- p + H_2$)*** and includes effects of the thermal motions of the gas molecules. The program has given good fits to $\mu^- p$ time distributions from experiments****
- The authors find that at least a 20% laser excitation is required, hence the laser should have at least one order of magnitude higher power.

Simulations different approach, more laser power



A third one confirming the previous

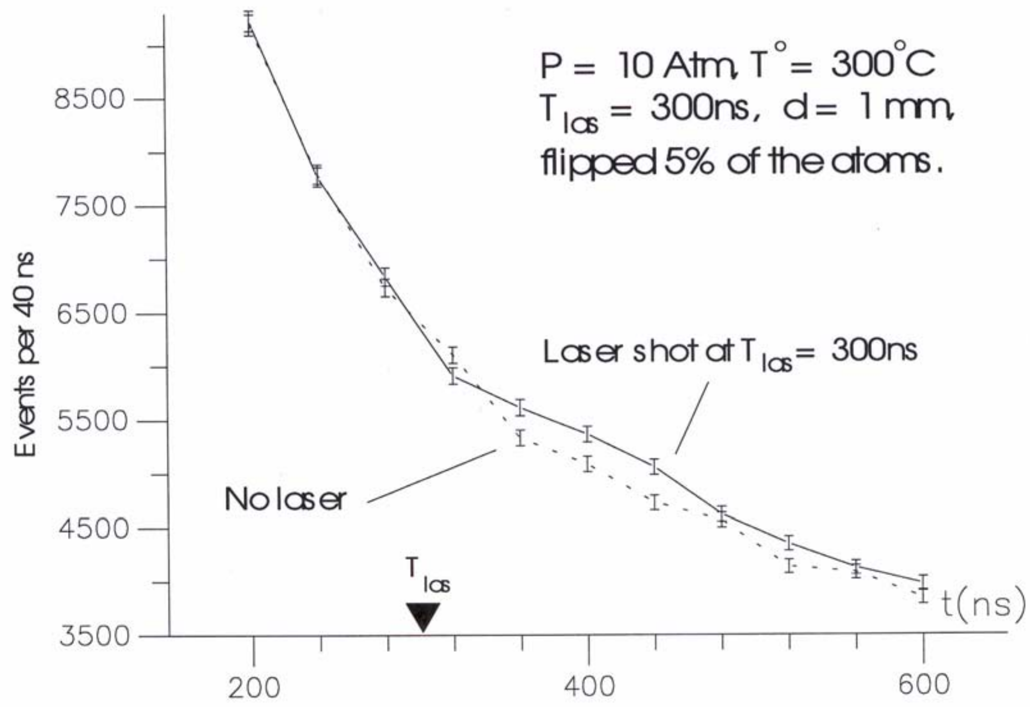
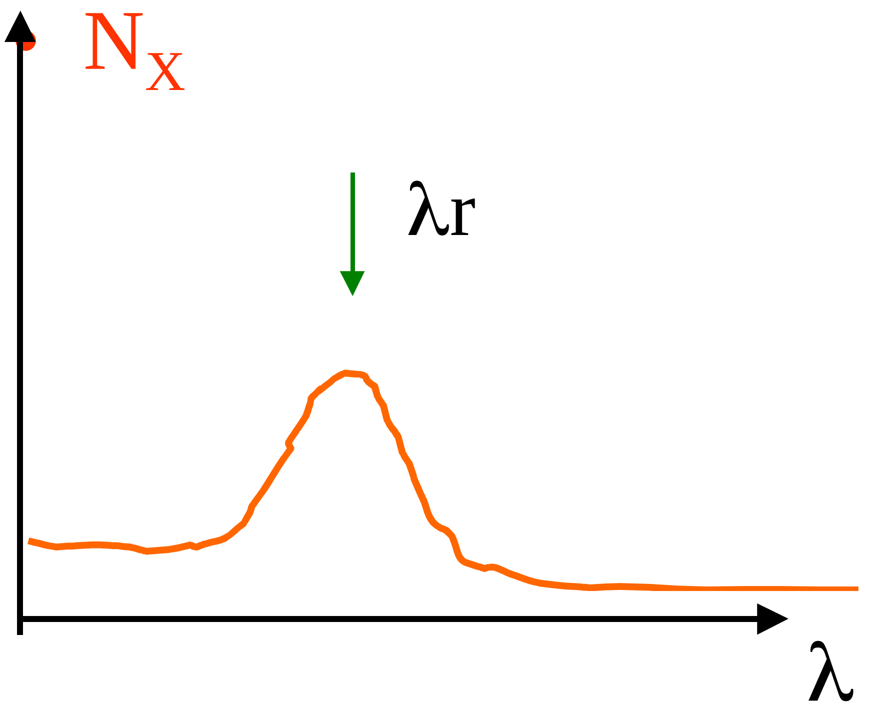


Fig.3. Time distribution of the events of Coulomb capture of 500000 thermalized ($p^-\mu$)-atoms by the plates spaced at d=1mm, at pressure P=10Atm and temperature T°=300°C. The solid line represents the curve obtained when a laser pulse of length 50ns is shot at T_{las}=300ns, and the spins of 5% of the ($p^-\mu$)-atoms in the target are flipped; the dashed line corresponds to the case when no laser pulse is shot.

The observable

- By counting the characteristics X rays with and without laser in a time window after t_{laser} and tuning the laser frequency around resonance one gets



5/1000000 ????

$\lambda = 6.62 \times 10^{-6}$ m

$\sigma = 6.62 \times 10^{-6} \times 5\ 10^{-6}$

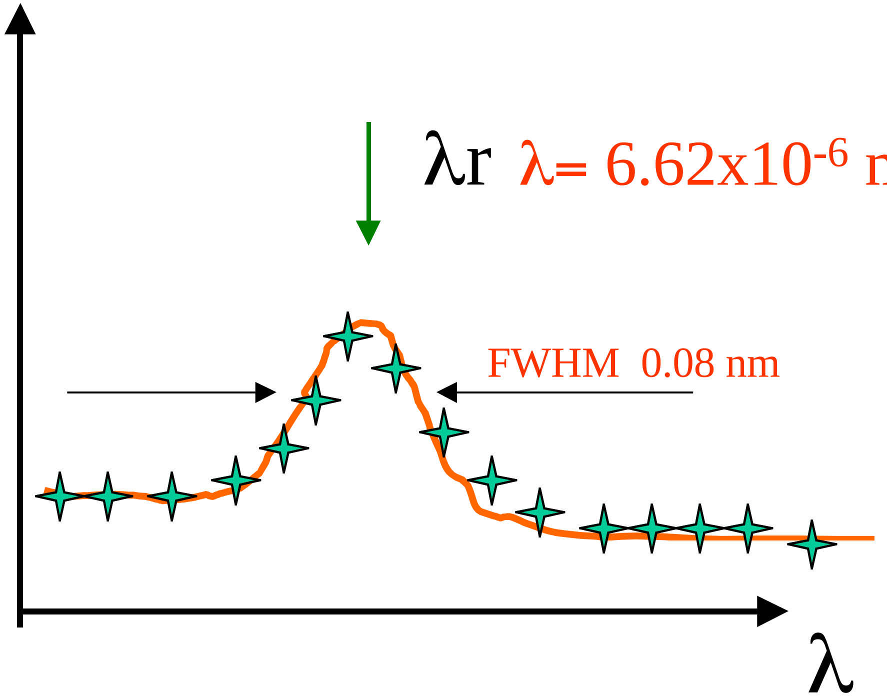
$\sigma = 33.1 \times 10^{-12}$ m

We require a precision on the determination of λ of about 0.03 Nanometers,

$\lambda = 6.62 \times 10^{-6}$ m

$\Delta\lambda = 0.033 \times 10^{-9}$ m

σ= fwhm/2.35 => fwhm= 0.08 nm



$\Delta E/E = 0.08 \times 10^{-9} / 6.6 \times 10^{-6} = 1.2 \times 10^{-5}$

DFB lasers line-width 0.01 = meV in single mode laser, $\Delta E/E = 5\ E^{-5}$

Line width is substantially determined by the Doppler effect at T= 273 K, and taking for the μp mass 10^9 eV

$$\frac{3}{2}KT = \frac{1}{2}m\beta^2$$

$$KT = \frac{1}{38}eV \rightarrow at \approx 300^0 K$$

$$m\beta^2 \approx \frac{1}{10}$$

$$\beta^2 \approx \frac{1}{10} \bullet \frac{1}{m} \approx \frac{1}{10} \bullet \frac{1}{10^9} \approx 1^{-10} \Rightarrow \beta \approx 10^{-5}$$

$$\Delta\lambda \approx \beta \bullet \lambda_0$$

$$\lambda_0 = 44THz \Rightarrow \Delta\lambda = 10^{-5} \bullet 44 \bullet 10^{12} = 440MHz$$

$$\Delta\lambda = 440MHz$$

$$\beta \approx 10^{-5}$$

$$\Delta\lambda \cong 10^{-5} \times \lambda_0 \cong 7 \times 10^{-6} \times 10^{-5} = 7 \times 10^{-11} m = 7 \times 10^{-2} nm$$

- QCL should be able to give a few Watt on an area 20 micron squared.
- 20 MW/cm2.
- The stop volume for the muons can be a cylinder of a basis of 1 cm diameter
- About the tunability, since the natural line width Δλ is totally induced by the Doppler effect, 1/10 of Δλ (supposing to scan the line in 10 steps), corresponds to 7×10^{-3} nm

$$\beta \approx 10^{-5}$$

$$\Delta\lambda \cong 10^{-5} \times \lambda_0 \cong 7 \times 10^{-6} \times 10^{-5} = 7 \times 10^{-11} m = 7 \times 10^{-2} nm$$

Natural line width and required precision

The Doppler width appears to be of the order of magnitude of the required precision

$\Delta\lambda_{doppler} = 0.07 \times 10^{-9}$ m

$\Delta\lambda_{expt} = 0.033 \times 10^{-9}$ m

We need a laser with $\Delta\lambda_{laser} = \Delta\lambda_{doppler} / 10$

$\Delta\lambda_{laser\ width} = 0.007 \times 10^{-9}$ m

Sweeping with the laser on a window of about

$\Delta\lambda_{sweep} = \Delta\lambda_{nat} \times 10 = 0.7 \times 10^{-9}$ m

Some example from the literature

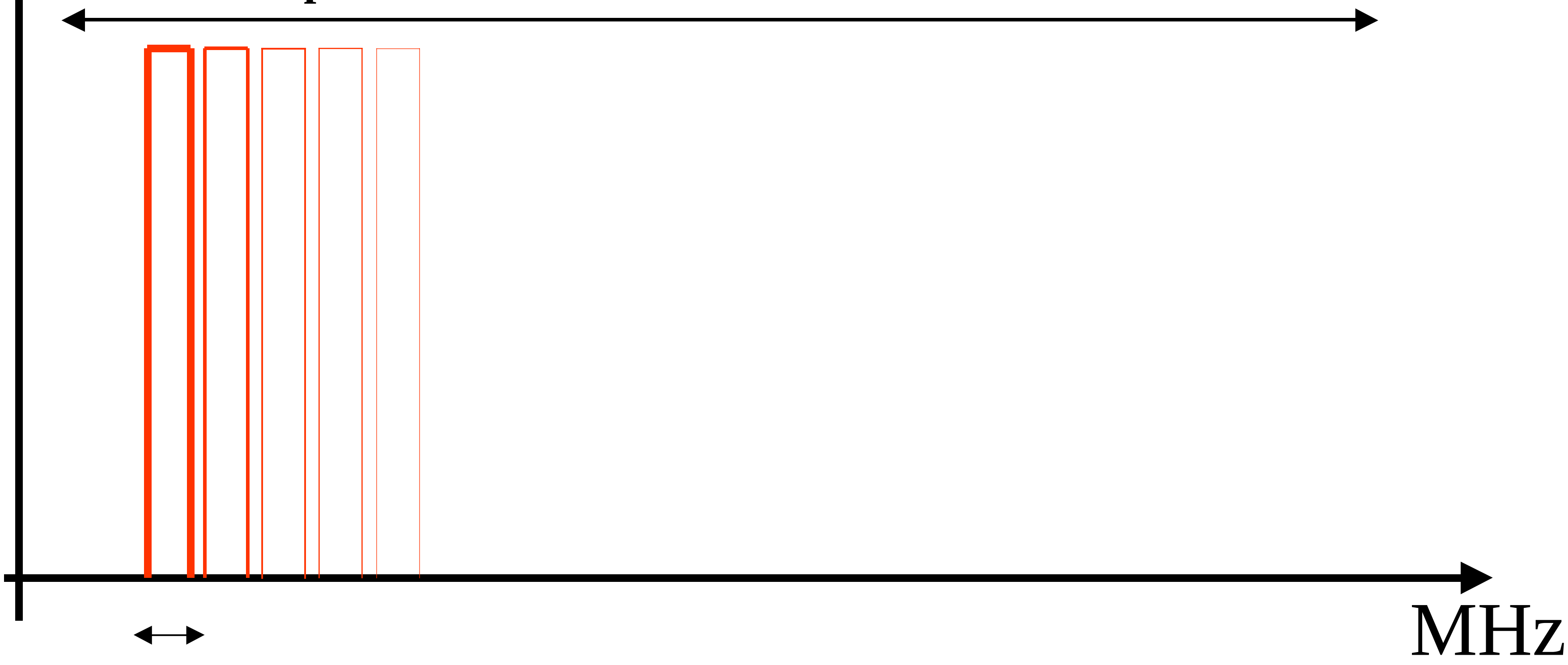
	Wavenumber k (cm^{-1})	Δk (cm^{-1})	λ (m)	E (eV)	ΔE (eV)	ΔE/E
1	965	0.25	10.3626943	0.119809439	0.000031039	2.59E-04
2	1898	0.15	5.2687039	0.235645923	0.000018623	7.90E-05
3	992.4	0.02	10.076582	0.123211283	0.000002483	2.02E-05
4	645.44	0.007	15.4933069	0.080134513	0.000000869084	1.08E-05

frequency k (cm^{-1})	Δυ (MHz)	λ (m)	E (eV)	ΔE (eV)	€
645.44	200	15.4933069	0.080134513	0.000000828	1.03E-05

Sweeping continuously requires good temperature or laser frequency CONTROL

ΔT=1K one gets $\Delta\lambda$ = 0.2 nm

$\Delta\lambda_{sweep}$=0.7nm, about 4K



50 MHz=0.007nm, about 40mK, $\Delta T/T$ about 1%

The transition probability is given by

$$P = \frac{\pi \mu_0}{4\hbar c} \lambda(\omega_0) \tau \left(\frac{e\hbar}{2M_\mu} \right)^2 |\mathbf{S}|_{\rm av}$$

A constant and three terms :

Laser frequency window $\lambda\,(\theta_0)$

Laser pulse length $\tau = 100$ **ns**

Laser mean power **S = 10 MW / cm** ^{2}

The constant numerical coefficient K is

$$(\pi \mu_0 / 4\hbar c)(e\hbar/2M_\mu)^2 = \alpha(\pi \hbar c)/2M_\mu c^2)^2 = 0.63 \cdot 10^{-27}\,{\rm cm}^2$$

For the laser line let's take a rectangular profile Γ = 50 MHz

$$\Gamma \approx 50\,MHz \approx 2 \bullet 10^{-7}eV \approx 3.2 \bullet 10^{-26}J$$

$$\lambda(\omega_0) = {}^{1}\!/_{\Gamma} = 0.3 \bullet 10^{26}J^{-1}$$

$$P = K \bullet \frac{1}{\Gamma} \bullet |S| \bullet \tau = 0.63 \bullet 10^{-27}cm^2 \bullet 0.3 \bullet 10^{26}J^{-1} \bullet 10^{7}\, {}^{J}\!/_{cm^2 s} \bullet 10^{-7}s$$

$$P = 0.02$$

Let us also suppose that we can build a cavity allowing about 10 reflections R = 10 => P x R = 20 %

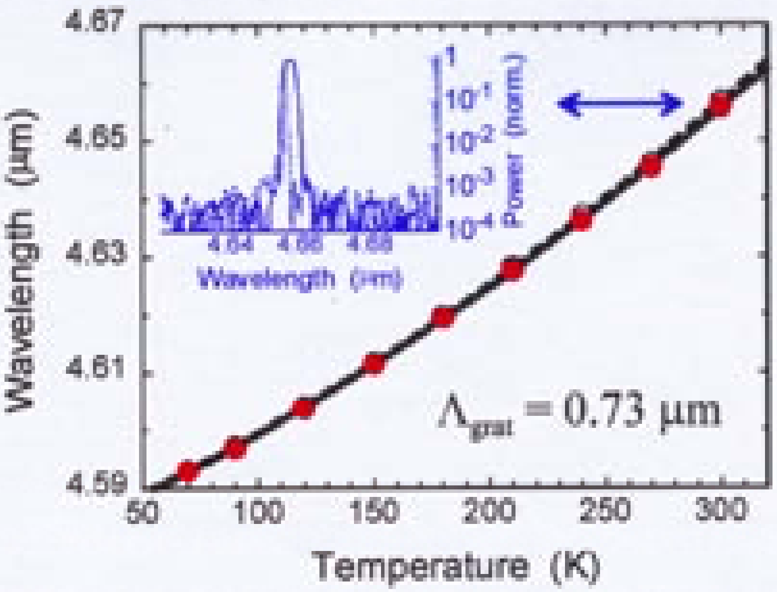
Every parameter is critical $\lambda(\theta_0)$, τ, S

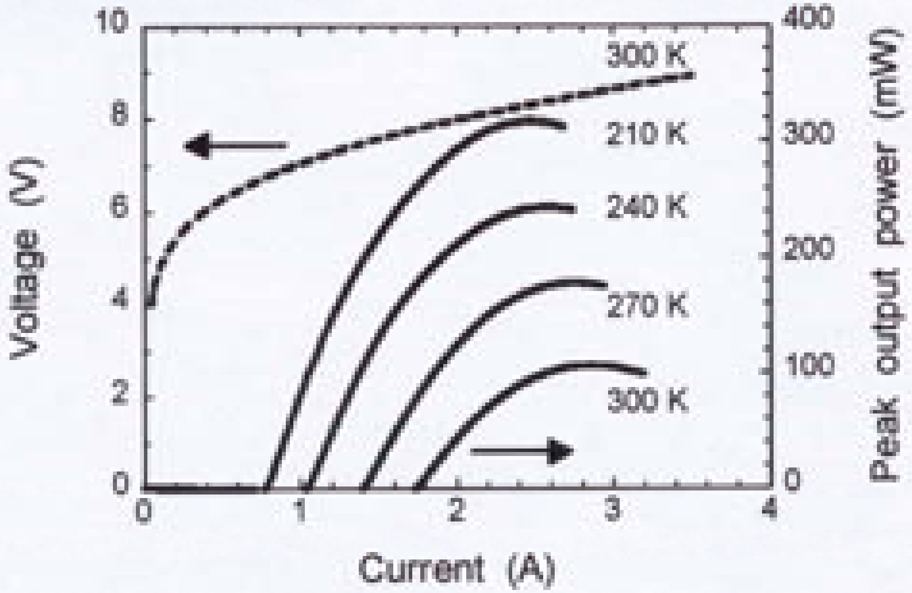
The dissipated power for a pulse of 100 ns

- We are hoping for power of 20x 10^6 W on a cm^2 during something like 100 ns, so we inject in the target

$$1 \times 10^{-7} \times 20 \times 10^{6} = 2\ W/cm^{2}$$

Room temperature, pulsed, single-mode QC-DFB laser @ λ ~ 4.6 μm





- 65 nm continuous single-mode tuning range with heat sink temperature
- high, single-mode, optical peak power (~ 100 mW) at room temperature

Lucent Technologies
Bell Labs Innovations

The line of the transition to be analysed in the experiment is at
6.7 μm =about 60000 A = 44 THz = .18 eV,

the Doppler width is 10^{-5} of this i.e.
6 x 10^{-5} μm = 0.6 A= 0.06 nm =440 MHz

the line width of the solid state laser could be of
50 MHz=0.007nm
λDoppler/λ laser= about 10

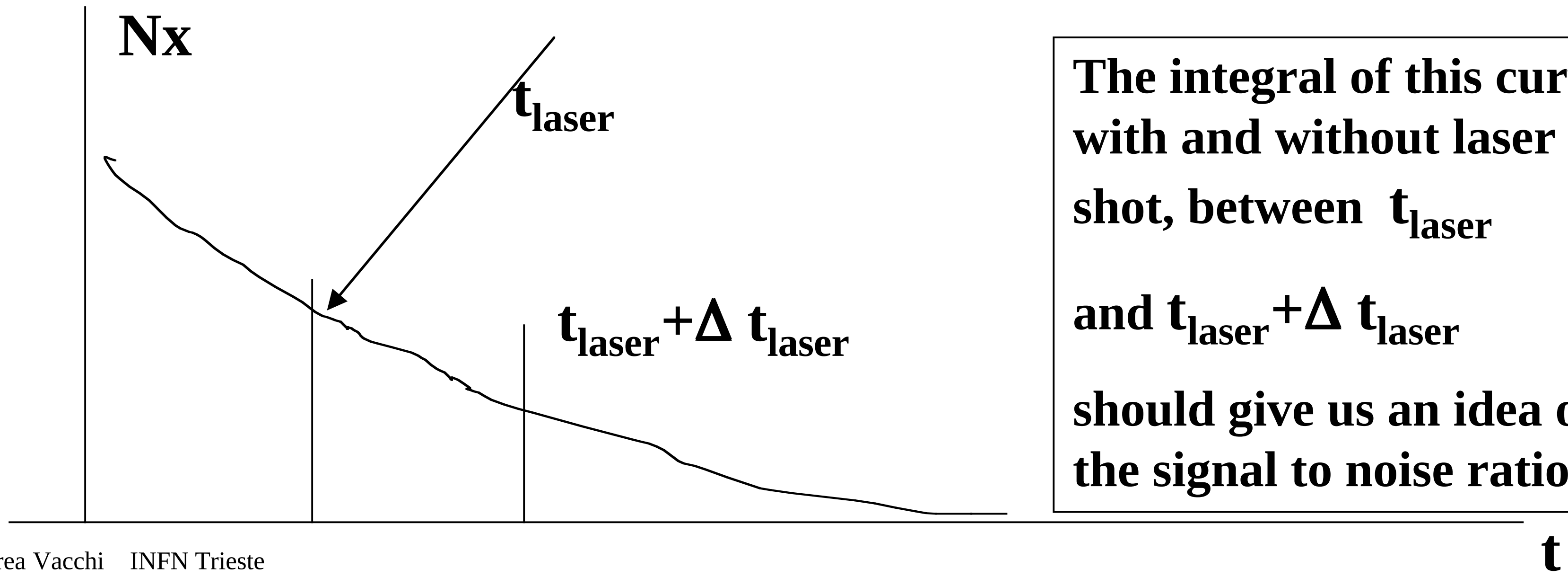
the scan should probably be on 10x λnatural=100 measuring points
$\Delta\lambda_{sweep}$= 0.7nm, about 4K
we need to control T to about 40mK

Let's try to see some more detail about the experiment

- Supposing
  - To have a bunched muon beam able to stop in the target 10^{-6} muons per bunch
  - That all 10^{-6} muons form μp atoms
  - That we have 100% efficiency in collecting the x ray from the transfer of the muons to the gold foils
  - the life time of the muonic atom in the target equals the life time of the muon = 2.2 μs

$$N_{stop} = 10^6 ; \quad N_x(t) = N_{stop}\, e^{-t/\tau}$$

- τ is the life time of the muon
- t is the time at which we measure Nx(t) x rays from muon transfer



The integral of this curve with and without laser shot, between t_{laser} and $t_{laser} + \Delta\, t_{laser}$ should give us an idea of the signal to noise ratio

Let first evaluate the no laser situation i.e. the back ground

$$N_{BG} = \left(N_{stop} \bullet e^{-t/\tau} - N_{stop} \bullet e^{-(t+100)/\tau} \right)$$

$$N_{BG} = N_{stop} e^{-t/\tau} \left(1 - e^{-100/\tau} \right) = 28000$$

- Let's suppose to allow for 7% of the atoms to be excited to the triplet state.

With the laser we allow for 7% of the atoms to be excited

we assume to have the laser shot at t_1=300 ns

$$N(t_1) = N_{stop} e^{-t_1/\tau} \bullet 7 \bullet 10^{-2} = 61000$$

$$N_{BG} / N_{Laser} = 0.49$$

- With a curve with 6 to 10 samplings we should do the job

Summary

- An experiment is proposed to measure the hyperfine splitting of the ground state of muonic hydrogen atom (μ^-**p**). In brief negative muons are to be stopped in hydrogen gas between Au foils placed about 1 mm apart in a target chamber, (gas pressure and temperature in the target have to be optimized 10 bar and 300K are used in simulations). Muons stopped in the gas under these conditions, form (μ^-**p**) with an initial energy of about 2 eV. The atoms rapidly (<ns) deexcite to the 1S ground state in a statistical mixture of spin states. As the atoms then diffuse within the gas to the Au foil surfaces, they undergo collisions with the gas molecules this cause the muonic atoms to be deexcited to the singlet state and thermalized within 100-200 ns. The time at which each (μ^-**p**) surviving muon decay reaches a foil surface is recorded by means of a muonic Au x ray emitted when the muon is transferred from the (μ^-**p**) to an Au atom. A large fraction of the (μ^-**p**) will reach the foil within 2 μs.
- A laser pulse will be applied to the gas target about 300 ns after the muons stopped. The laser will excite a fraction of all the (μ^-**p**) atoms to the triplet state. Through collisions the (μ^-**p**) will rapidly return to the singlet state with a average kinetic energy gain of about .12 eV. This will affect the the time distribution of the (μ^-**p**) arrivals at the foils by boosting the number of arrivals shortly after the laser pulse with a compensating depletion of the number of arrivals at later times.

Collection of references

- 1) S.D.Drell and J.D. Sullivan. Phys. Rev. **138** (1965) B446.
- 2) E.deRaphael. Phys. Lett. **37B** (1971)201.
- 3) S.J. Brodsky, S.D.Drell . Ann. Rev. Nucl.Sci. **20**(1970)147.
- 4) G.T. Bodwin, and D.R. Yennie, Phs. Rew., D: Part. Fields, 1988, vo 37, p. 498.
- 5) S.G. Karshemboim, Phys. Lett. A, 1997, vol.225,p.97.
- 6) A.C. Zeemach. Phys Rev. 104 (1956) 1771
- 7) C.K. Iddings, P.M. Platzman. Phys Rev. 113 (1959) 192
- 8)V.W.Hughes and J. Kuti Ann. Rev. Nucl. Part. Sci. **33** (1983) 611
- 9) G.T. Bodwin, D.R. Yennie, M.A. Gregorio. Rev. Mod. Phys. **57** (1985) 723.
- 10) A.P. Martynenko, R.N. Faustov, Physics of Athomic Nuclei, Vol 61, No.3, 1998, pp 471-47
- 11) V.W.Hughes and J. Kuti Ann. Rev. Nucl. Part. Sci. **33** (1983) 611

¥ 12) AAdamczak, DBackalov, EZavattini

- 13) K.Kato IEEE Journ.Quant. Electr. **QE-20** (1984) 698.
- 14) G. Carboni et. al. Nucl.Phys. **A278** (1977) 381.
- 15) A. Bertin et al., Phys.Lett. B 8 (1978) 355.
- 16) L.I. Ponomarev et al., Sov. J. Nucl. Phys. 29 (1979) 67.
- 17) Bacalov D., Milotti E., Rizzo C., Vacchi A., Zavattini E., Phys. Lett. A172, 277-280 (1993).

Muonic hydrogen
hadronic vacuu m polarization

- When the electron is replaced by a muon, the relative contribution of the various effects to $\Delta E^{hfs}_{\exp}$ change, the contribution of hadronic vacuum polarization, omitted for hydrogen because of his smallness, may become more significant. Recent calculations* shows how ΔE^{hfs}_{h} =861.05 MHz, this contribution, determined primarily by the pion form factor, is due the increase in the relative momentum of the particles which reaches values on the order of the hadron masses. The theoretical error does not exceed 10%

Physics Letters A 172 (1993) 277–280
North-Holland

PHYSICS LETTERS A

Experimental method to measure the hyperfine splitting of muonic hydrogen $(\mu^- p)_{1S}$

D. Bakalov [1], E. Milotti, C. Rizzo, A. Vacchi and E. Zavattini
Dipartimento di Fisica dell'Università di Trieste, via Valerio 2, Trieste 34017, Italy
and Sezione INFN di Trieste, Area di Ricerca, Padriciano 99, Trieste 34012, Italy

Received 31 July 1992; revised manuscript received 17 October 1992; accepted for publication 8 November 1992
Communicated by B. Fricke

We propose an experimental method to measure the hyperfine splitting of the energy level of the muonic hydrogen ground state $(\mu^- p)_{1S}$ by inducing a laser-stimulated para-to-ortho transition. The method requires an intense low energy pulsed μ^- beam and a high power tunable pulsed laser.

1. Introduction

In the present paper an experimental method to measure the hyperfine splitting ΔE^{hfs} in the muonic atom $(\mu^- p)_{1S}$ is proposed (see fig. 1) and the results of a Monte Carlo simulation are presented. We believe that an accuracy of 10 ppm can be reached with a sufficiently intense pulsed tunable infrared laser [1,2]; the results of such an experiment would be complementary to the very precise experimental result of the hyperfine splitting of the ground state of the hydrogen atom [3],

$$\Delta E^{hfs}_{exp} = 1420.4057517864(17)\ \text{MHz}\,. \quad (1)$$

[1] On leave from: Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Blvd. Trakia 72, Sofia 1784, Bulgaria.

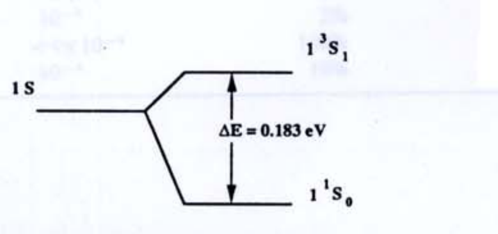


Fig. 1. Hyperfine structure of the ground state of muonic hydrogen $(\mu^- p)_{1S}$. The nonrelativistic level 1S is split into a triplet $1\,^3S_1$ and a singlet $1\,^1S_0$ hyperfine state.

The theoretical expression for the hyperfine splitting ΔE^{hfs}_{th} of the hydrogen-like atom $(\ell^- p)_{1S}$ (ℓ^- standing for μ^- or e^-) can be written as [4]

$$\Delta E^{hfs}_{th} = \Delta E^{F}(1 + \delta^{QED} + \delta^{FF} + \delta^{POL})\,, \quad (2)$$

with the Fermi formula

$$\Delta E^{F} = \tfrac{8}{3}\alpha^4 c^2 \frac{\mu_p m_p^2 m_\ell^2}{(m_p + m_\ell)^3}\,, \quad (3)$$

where m_p is the mass of the proton, m_ℓ the mass of the lepton, α the fine structure constant, μ_p the proton magnetic moment and c the velocity of light in vacuum.

The term δ^{QED} is the contribution from the higher-order quantum-electrodynamical effects; it is known to better than 1 ppm [5] and, in principle, the calculation can be improved. The terms δ^{FF} and δ^{POL} describe the contribution due to the internal structure of the proton; of course, these terms vanish for point-like "nuclei" (like in the case of muonium and positronium [6]). The term δ^{FF} depends, in first approximation, on the rigid spatial distribution of the proton charge and magnetic moment, associated to the proton electromagnetic form factors as obtained from e–p elastic scattering data. This term can be expressed [7], in the lowest approximation, as

$$\delta^{FF} = -2[m_\ell m_p \alpha c/\hbar(m_\ell + m_p)]\langle R_{pr}\rangle\,, \quad (4)$$

where $\langle R_{pr}\rangle$ is the integral of a combination of the

0375-9601/93/$ 06.00

Pulsed muon Beam

- Using a pulsed muon source makes considerable advantage since the laser is easier to synchronize. The reference is the number of μ^-/ s stopped in the given volume.
  - The stop density at the existing muon beam at PSI is of the order of 10^6 $\mu^-/(\mu s)$.
  - As an example the beam structure at Rutheford RAL (pulse/sec ~100 ns long) is well suited for laser operation, however the mean muon intensities are ~100 times lower than at PSI.