Link between Spin Statistics Connection and Cosmic Gravity?

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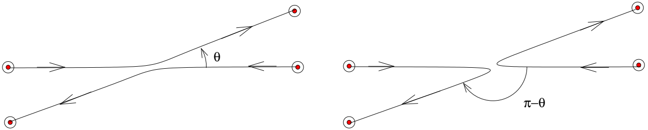
Spin-Statistics Connection

- Spin-Statistics Connection:
  Integer-spin identical particles are bosons (B-E statistics).
  Half-odd-integer spin identical particles are fermions (F-D statistics).
- Operationally: Two identical spin-s particles scattering: both spins perpendicular to plane of scattering.

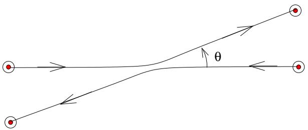


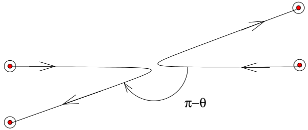
- If s is integer/half-odd-integer, add/substract scattering amplitudes of the two processes.
- Proofs of SSC are kinematic mostly as a consistency condition in rel. QFT.

Spin-Statistics Connection from Dynamics?

- A dynamical reason for SSC?
- If so, gravitational as gravity is the only known force coupling to all particles.
- In electrodynamics, all magnetic moments / magnetic effects are due to electric currents.
- Similarly taking spin as a matter current, obtain all spin related effects from coupling to matter currents?
- Try to get the relative $\pm$ sign from some spin-dependent dynamics.
- Independent of the type of scattering interaction.

Dynamical phase required to get the relative sign





- A particle's momentum changes through angle θ at some angular velocity ω.
- Consider dynamical interaction $H_{int} = -s\omega$.
- Each particle picks up a phase $\exp(i \int s\omega dt) = \exp(is\theta)$ in the first process, and $\exp(is(\theta - \pi))$ in the second.
- We get a relative phase between the processes $\exp(i2s\theta - i2s(\theta - \pi)) = \exp(is2\pi)$ i.e. ± 1 depending on s being integer/half-odd-integer.

Particle's frame

- A particle's momentum changes through angle θ with respect to the cosmic frame (at some angular velocity ω).
- Friedmann-Robertson-Walker metric with flat spatial slices $ds^2 = -dt^2 + a(t)(dx'^2 + dy'^2 + dz'^2)$.
- Transform to frame of particle (t, x, y, z): $x' = x\cos\omega t - y\sin\omega t,\ y' = x\sin\omega t + y\cos\omega t$.
- $$g_{\mu\nu} = \begin{pmatrix} -1 + \left(x^2 + y^2\right)\omega^2 & -y\omega & x\omega & 0 \\ -y\omega & 1 & 0 & 0 \\ x\omega & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
  set $a(t_{now}) = 1$.
- Off-diagonal components $\Longrightarrow$ gravitomagnetic vector potential due to matter current in particle's frame.

GravitoElectroMagnetism

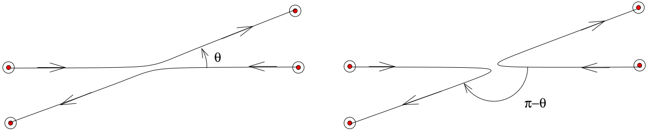
- Linearized approach GEM: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu} \ll 1$.
- $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ where $h = \eta^{\mu\nu}h_{\mu\nu}$.
- GEM potentials $4\Phi = \bar{h}_{00}$ and $-2A_i = \bar{h}_{0i}$, $A_i = (\frac{1}{2}y\omega, -\frac{1}{2}x\omega, 0)$ (set $a(t_{now}) = 1$).
- In particle's frame, a rotating cosmic matter/energy current produces a gravitomagnetic field $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} = (0, 0, -\omega)$.
- $\boldsymbol{\mu} = q\mathbf{s}/2mc$, with q replaced by $-2m$ gives $\boldsymbol{\mu_g} = -\mathbf{s}$ (set $c = 1$).
- The field interacts with the spin just as in electrodynamics $H_{int} = -\boldsymbol{\mu_g}.\mathbf{B}$
- $H_{int} = -s\omega$.
- Exact result as this gravitomagnetic $\boldsymbol{B}$ gives the Coriolis force as a Lorentz force (talk by C.S.Unnikrishnan).

SSC via Khriplovich Hamiltonian

- Khriplovich[1] gives a convenient expression (in tetrad components except u^0_w) for $H_{int} = s^i \epsilon_{ikl}(\frac{1}{2}\gamma_{klc} + \frac{u^k}{u^0+1}\gamma_{0lc})\frac{u^c}{u^0_w}$.
- e^μ_a are the tetrads of the particle's frame.
  $e^\mu_0 = (1, \omega y, -\omega x, 0) = (\frac{\partial}{\partial t} + \omega y \frac{\partial}{\partial x} - \omega x \frac{\partial}{\partial y})$, $e^\mu_1 = (0,1,0,0) = \frac{\partial}{\partial x}$, $e^\mu_2 = (0,0,1,0) = \frac{\partial}{\partial y}$, $e^\mu_3 = (0,0,0,1) = \frac{\partial}{\partial z}$.
- $u^c = (\gamma, -\gamma\omega y, \gamma\omega x, 0)$ is the 4-velocity of the particle.
- $u^0_w = \gamma$ (relativistic factor) is a coordinate component of the 4-velocity.
- $\gamma_{abc} = e_{a\mu;\nu} e^\mu_b e^\nu_c$ are the Ricci rotation coefficients
- $\gamma_{1,2,0} = -\omega$, $\gamma_{2,1,0} = \omega$ (others zero).
- $H_{int} = -s\omega$.

[1] I.B. Khriplovich, *Spinning relativistic particles in external fields*, Acta Physica Polonica B Proceedings Supplement, **1**, 197, 2008.

SSC from Gravity



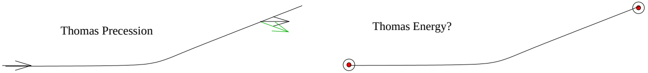
- Each particle picks up a phase $\exp(-i \int H_{int}\, dt) = \exp(is\theta)$ in the first process, and $\exp(is(\theta - \pi))$ in the second.
- This is a dynamical gravity dependent phase over and above the phases due to scattering interaction.
- We get a relative phase between the processes $\exp(i2s\theta - i2s(\theta - \pi)) = \exp(is2\pi)$ i.e. ± 1 depending on s being integer/half-odd-integer.
- Proposed in C S Unnikrishnan, arXiv:gr-qc/0406043 v1, 2004.

Merits of the proof

- Dynamical proof based on cosmic gravity.
- Independent of scattering interaction.
- Positive proof i.e. goes beyond showing the impossibility of 'wrong statistics' for given spin.
- Applicable to any spin.
- Also shows why rotation based exchange proofs seem to work.

Inconsistent?

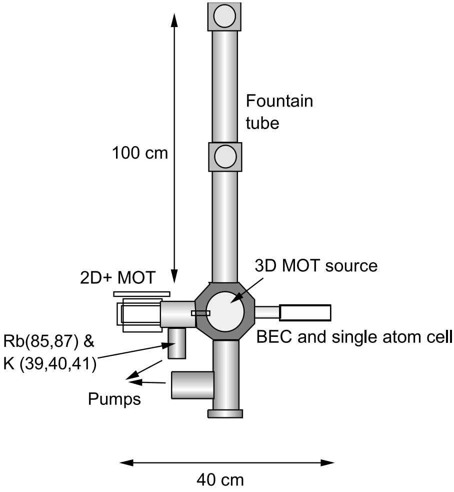
- Essential point for SSC: the phase picked up by a spin-s particle moving at angular velocity ω should be $\exp(-i\int H_{int}dt)$ with $H_{int} = -\mathbf{s}.\boldsymbol{\omega}$.
- Like spin-orbit coupling so compare with spin-orbit coupling in atom.



- Spin of electron rotating at ang. freq. ω precesses $d\mathbf{s}/dt = \mathbf{s}\times\omega_T$ at average (round trip) ang. frequency $\omega_T = (1-\gamma)\boldsymbol{\omega}$: Thomas precession.
- Electrons in an atom, due to orbital motion at ang. frequency ω (spin-orbit coupling), pick up a Thomas correction to the energy $-\mathbf{s}.(1-\gamma)\boldsymbol{\omega} \simeq s\frac{1}{2}\frac{v^2}{c^2}\omega$ (manifests as fine-structure energy splitting between spin up and spin down states: $2s\omega_T$).
- NOT $H_{int} = -\mathbf{s}.\boldsymbol{\omega}$ (as needed for SSC)!!!

Experiment (Under Construction)

- Two neutral K-40 (fermionic) atoms captured in a steep magneto-optic trap / electric dipole trap.
- Ultra high vacuum 10^{-11}Torr.
- Cooled ($\sim 1\mu$K) to ground state of the trap.



To Observe

- Ground state width of Gaussian wave function $\sim$ few μm.
- Inter-particle separation $\sim \mu$m.
- Optical resolution of imaging $\sim 1\mu$m.
- Spin-polarised atom-atom separation due to Pauli exclusion.

Future Directions

- The experimental fine structure splitting is seen as a consistency check for this proof.
- Try to get around the Thomas correction inconsistency to get Spin-Statistics Connection via gravitational dynamics.
- Side issue: Malykin (2006) and Ritus (2007) in Physics Uspekhi claim that Thomas precession frequency as seen in the lab frame is $\frac{\gamma-1}{\gamma}\omega$ and not the textbook result $(1-\gamma)\omega$: a sign difference and γ factor.
- Set up experimental capability to measure fermionic atom-atom correlations.