Spin-Statistics Violations from Superstring Theory

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MGJ, arXiv:0803.4472, arXiv: 0809.2733

MGJ and S. Hellerman, work in progress

Theoretical and Experimental Aspects of the Spin-Statistics Connection and Related Symmetries Workshop

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Introduction

- It is a well-established experimental fact that a particle's spin (integral or half-integral) and its statistics (symmetric or antisymmetric) are found to be correlated,

$$\text{bosons} : [a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k},\mathbf{k}'}, \qquad \text{fermions} : \{b_{\mathbf{k}}, b^\dagger_{\mathbf{k}'}\} = \delta_{\mathbf{k},\mathbf{k}'}.$$

- There are a variety of ways to modify these relations based on breaking of Lorentz invariance, locality, etc. (see review by Greenberg 2000)
- Any detected violations, however slight, would be tremendously important for physics
- Could even have cosmological consequences due to mismatch in loop cancellation of vacuum energy (MGJ and Hogan 2007)

Motivation

- Ideally some high-energy theory would predict exactly how SS violations would come about
- The leading such theory is <u>superstring theory</u>, which relies on extended objects (strings, membranes, etc.) and so in principle could easily produce such violations

Outline

- Basics of Gauge Theory and Superstring Interactions
- Heterotic Worldsheet Linkings
  - Motivation
  - Explicit Instantonlike Solutions
  - Violation in Effective Field Theory
  - Experimental Bounds
- Braneworlds and Noncommutative Geometry
  - Basics
  - Relationship between string theory and NCG
  - NCG and spin-statistics violations
  - Experimental Bounds

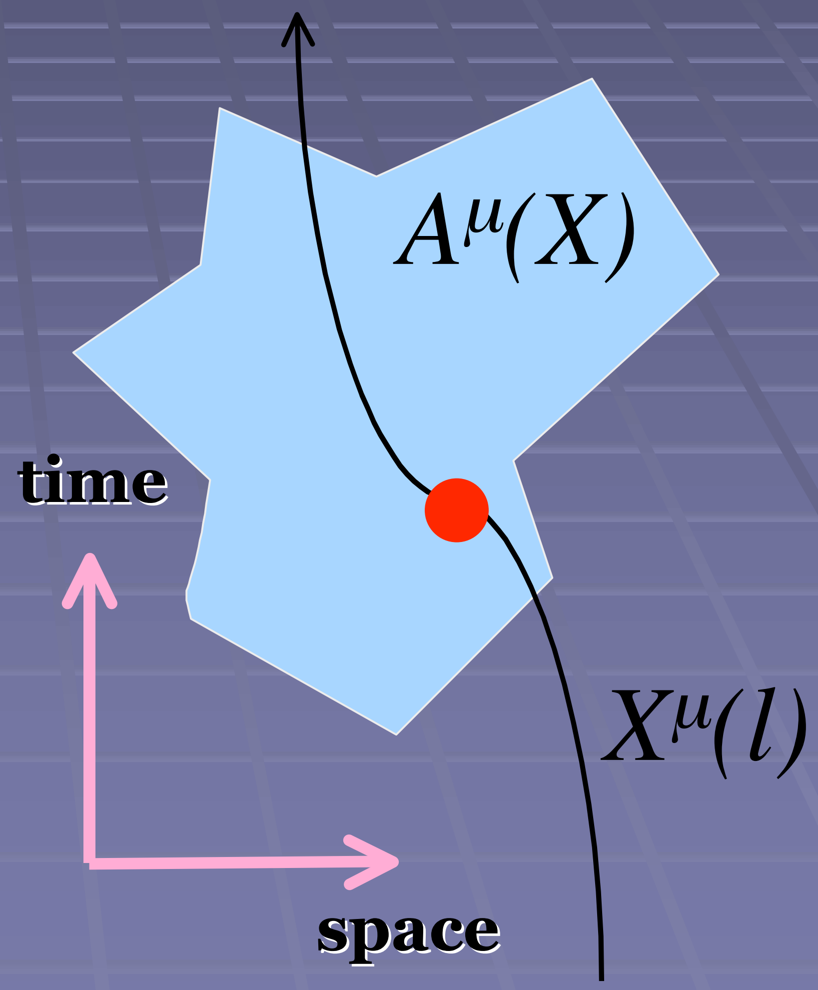
Gauge Theory Interactions

- Point particles couple to a 1-form gauge field A_μ via the worldline interaction term

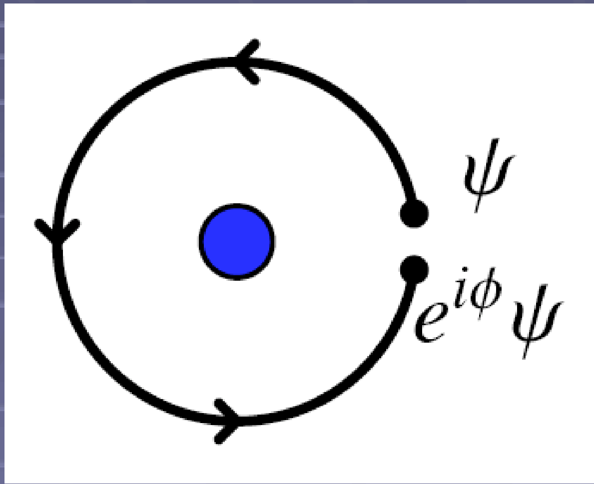
$$S = q \int dl \; \dot{X}^\mu A_\mu .$$

which is then used to compute amplitudes via the path integral

$$\mathcal{A}(\cdots) = \int [\mathcal{D}A]\,[\mathcal{D}X]\, e^{iS[A,X]} (\cdots)$$



Statistical Phases in 2+1 Dims



- **Consider a second particle producing a localized flux tube given by**

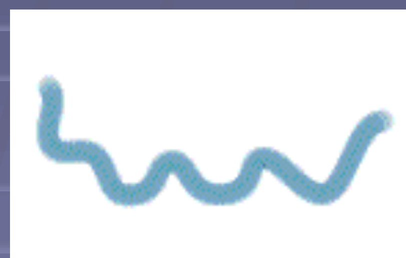
$$A_i = -\frac{\Phi \epsilon_{ij} X^j}{4\pi |X|^2}, \qquad B_{12} = \Phi \delta^2(X).$$

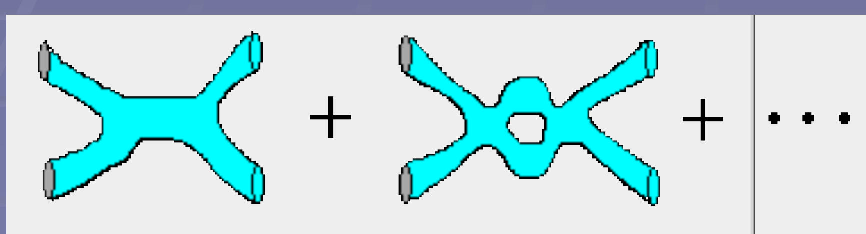
- **Moving one particle around another is a topologically well-defined process in 2+1 dimensions and with some coupling *q* to gauge field will produce a phase *a la* Aharonov and Bohm:**

$$\Delta\phi = q \int dl\ \dot{X}^i \left(-\frac{\Phi}{4\pi} \epsilon_{ij} \partial^j \ln |X| \right) = q\Phi.$$

- **Thus we can have particles of <u>any</u> statistics, named 'anyons' (Wilczek 1982)**

A Very Quick Summary of Superstring Theory





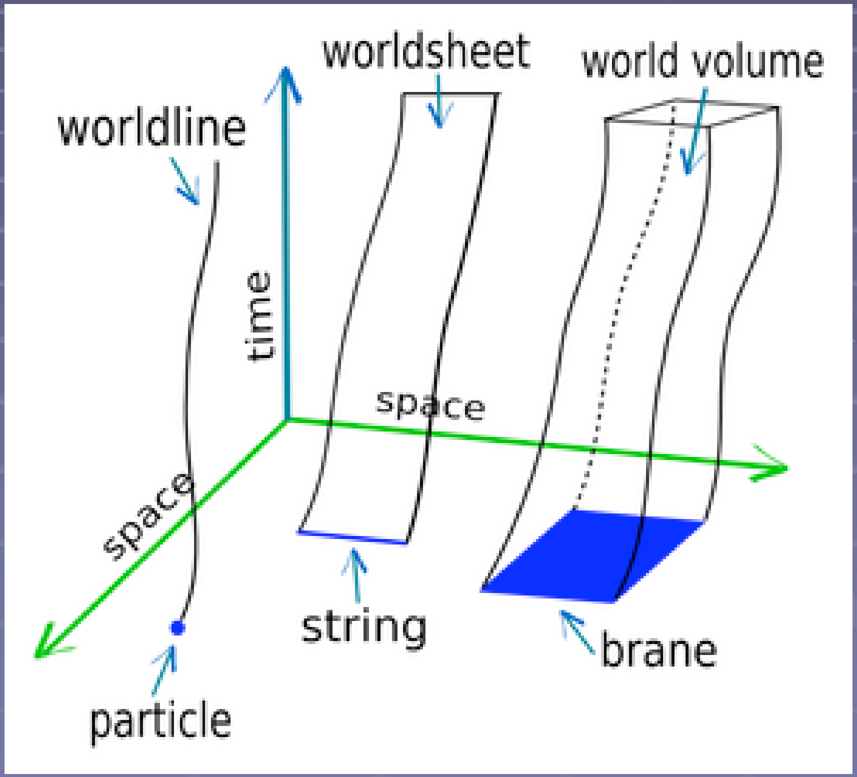
- Superstring Theory models all elementary particles as tiny vibrating strings
- The oscillations of the strings are in principle completely determined, and thus so is the spectrum
- We can also perform the path integral over the position of strings to compute scattering amplitudes

Worldsheet Interactions

- Similar to point particles, strings couple to a 2-form gauge field $B_{\mu\nu}$ via the worldsheet interaction term

$$S = \int d^2z \, \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}.$$

- We can use this in exactly the same way as for point particles!



Method #1: Phases in 3+1 Dims, 'linkings'

- Moving a particle through a loop of string is topologically well-defined in 3+1 dimensions and can produce a phase via an appropriate coupling
- Such a topological 'linking number' is defined as

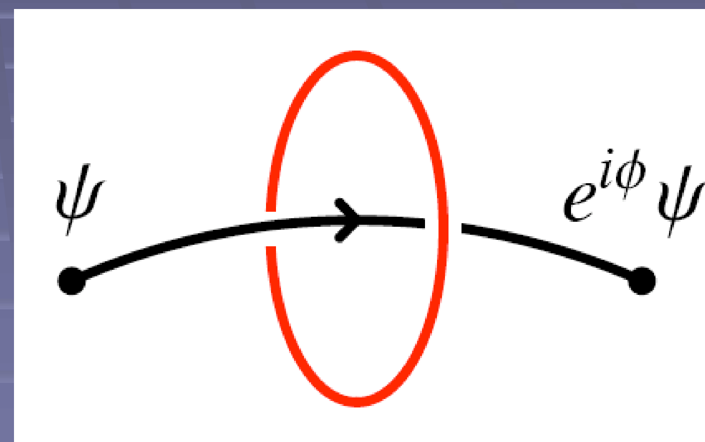
$$N = \frac{\epsilon^{\mu\nu\rho\lambda}}{4\pi^2}\int d\Sigma_{\mu\nu}(X)\int dY_\rho \frac{(X-Y)_\lambda}{|X-Y|^4}$$

- Comparing this to the previous coupling

$$S = \int d^2z\, \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}$$

this means we desire the second particle to source the *B*-field as

$$B_{\mu\nu}(x) = \frac{q\epsilon_{\mu\nu\rho\lambda}}{\theta}\int dl\, \partial^{[\rho}G(x-Y)\dot{Y}^{\lambda]}$$



Heterotic Worldsheet Linkings

- Such a sourcing can be obtained for a changed particle using the '*BF*' term

$$S_{BF} = \int d^4x \ \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} \partial_\rho A_\lambda$$

- This arises naturally in the context of heterotic strings, and so if we approximate one such string as pointlike we could produce a linking-induced phase as above
- This was utilized by Harvey and Liu 1990 to possibly produce small violations of spin-statistics.

A Paradox!?

- How can string theory, which has always produced local, Lorentz-invariant point-particle quantum field theories, give such a violation?

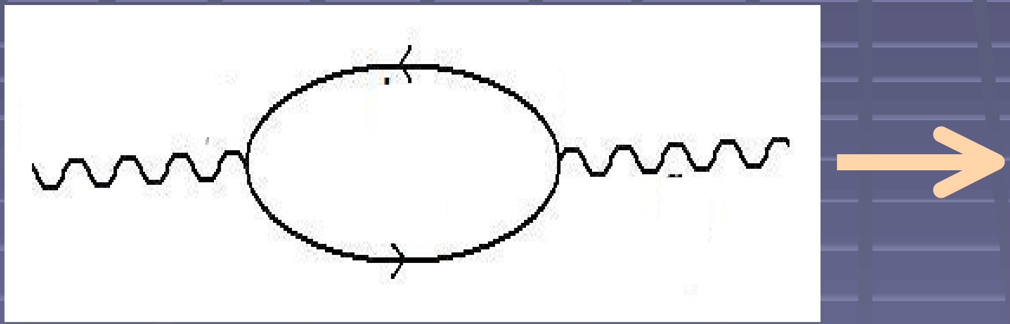
Non-local Propagators

- This type of interaction could only be modeled by having the usual spacetime propagator be modified into something of the form:

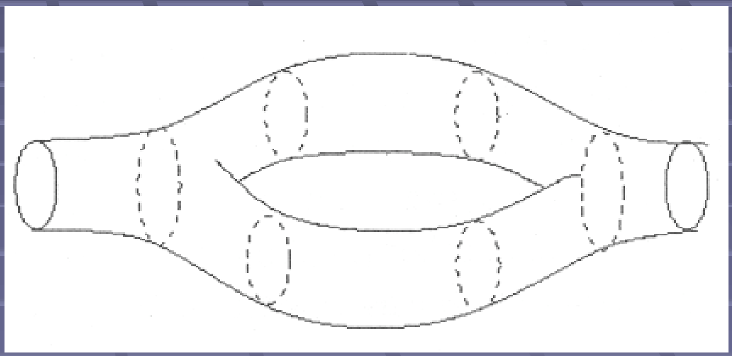
$$\text{bosons}: \frac{1}{(p^2 - m^2)^{1+\epsilon}}, \qquad \text{fermions}: \frac{\not{p} + m}{(p^2 - m^2)^{1+\epsilon}}, \qquad 0 < |\epsilon| \ll 1.$$

- Such 'nonlocal' propagators (since they form an infinite series in $p \sim -\, id/dx$) allow one to evade the Spin-Statistics Theorem (Gulzari, Srivastava, Swain, Widom 2006; da Cruz 2000, 2004)
- These have occurred previously in string theory, but only on strange backgrounds (Taylor and Zwiebach 2003)

1-loop Violation of Spin-Statistics

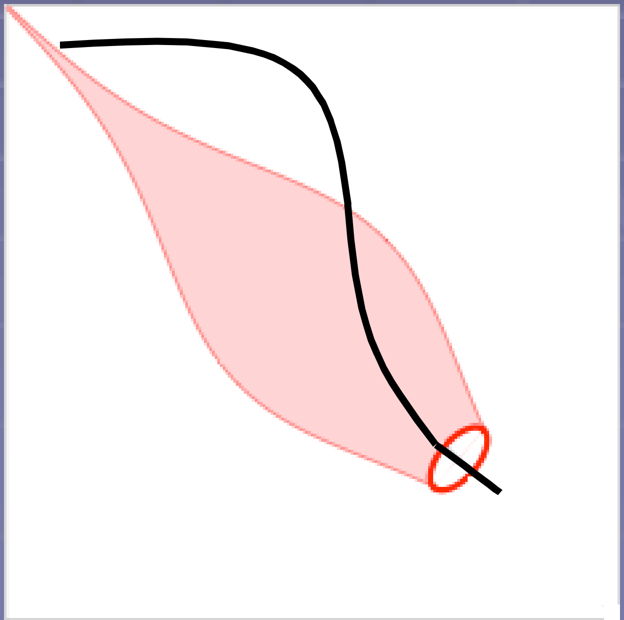


Worldsheet



- Such an effect is expected to appear from 1-loop perturbative corrections to the propagator, which for the worldsheet is topologically a torus.
- This corresponds to the string emitting and then absorbing a virtual photon which has passed through its worldsheet

Spacetime



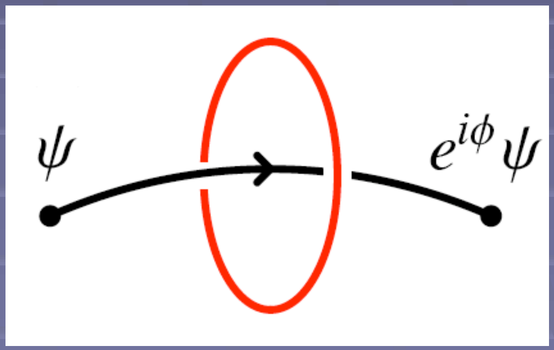
Harvey-Liu process whereby one string momentarily expands sufficiently to envelop another, producing a phase in the path integral.

How Large is this Effect?

- The magnitude of this effect is expected to be suppressed by the energy scale:

$$A \sim \exp(-\Delta x^2 / \alpha'),$$

$\Delta x \sim 1/E$



(where $1/\alpha'$ is the string tension)

- This unfortunately makes the effect extraordinarily difficult to observe: for a typical value of $\alpha' \sim (10^{16}\text{ GeV})^{-2}$, a ~ TeV string would have a violation of order ~ $\exp(-10^{26})$.

Evaluating the 1-loop Amplitude

- Investigation of this process is currently underway (MGJ and S. Hellerman). But in the meantime, maybe there is a simpler toy model which could estimate the importance of such an effect, that of worldsheet instantons.

Explicit Instanton Solutions

- Let us try to construct explicit solutions for these instantons (Jackson 2008). The action for the first (extended) string is

$$S_1 = \frac{1}{2\pi\alpha'} \int d^2z \left[\partial X^\mu \bar{\partial} X^\nu (\delta_{\mu\nu} + 2\pi\alpha' B_{\mu\nu}) + 2\pi\alpha' \delta^2(z,\bar{z}) k_1 \cdot X\right].$$

The action for the second (point-like) string is

$$S_2 = \int dl \left[\frac{1}{2\alpha'} \dot{Y} \cdot \dot{Y} + \dot{Y} \cdot (iqA - k_2)\right].$$

The action for the gauge fields is

$$S_{gauge} = \int d^4x \left[\frac{3\alpha'}{32g^2} \tilde{H}^2 + \frac{1}{4g^2} F^2 + \theta \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} \partial_\rho A_\lambda\right]$$

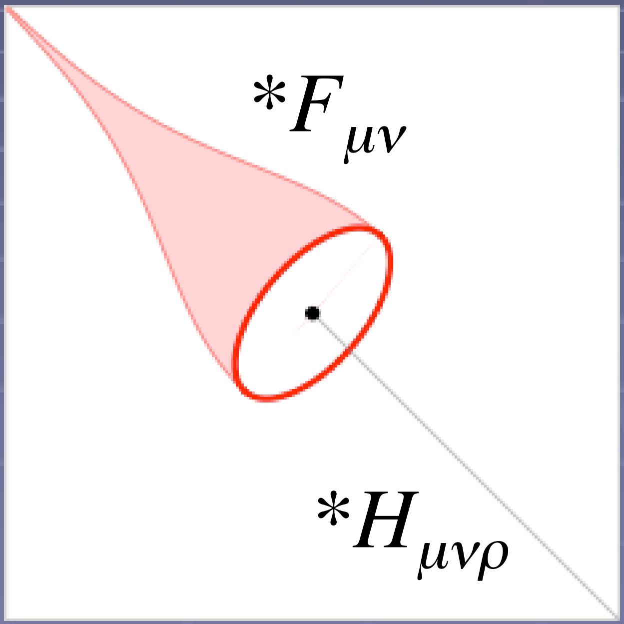
- By taking $\theta g^2 \to \infty$ then the gauge kinetic fields can be neglected and we can solve for *A,B* explicitly.

The *BF* coupling in a simplified limit

$$B_{\mu\nu}(x) = \frac{q\epsilon_{\mu\nu\rho\lambda}}{\theta}\int dl\ \partial^{[\rho}G(x-Y)\dot{Y}^{\lambda]},$$
$$A_{\mu}(x) = \frac{i\epsilon_{\mu\nu\rho\lambda}}{2\theta}\int d^2z\ \partial^{[\nu}G(x-X)\partial X^{\rho}\bar{\partial}X^{\lambda]}$$

$$S_{eff} \sim -\theta\int B\wedge F$$
$$\sim -\frac{iq}{\theta}\frac{\epsilon^{\mu\nu\rho\lambda}}{4\pi^2}\int d\Sigma_{\mu\nu}(X)\int dY_{\rho}\frac{(X-Y)_{\lambda}}{|X-Y|^4}.$$

This is the linking number!



The BPS Transformation

- We can 'complete the square' to rewrite this as

$$
\begin{aligned}
S_{eff} &= \frac{1}{2\pi\alpha'}\int d^2z\ |\partial(X-\alpha' k_1 \ln|z|)|^2 + \frac{1}{2\alpha'}\int dl|\dot{Y}-\alpha' k_2|^2 + i\frac{qN}{\theta} \\
&= \frac{1}{2\pi\alpha'}\int d^2z\ \Big|\partial(X^\mu - \alpha' k_1^\mu \ln|z|) \\
&\quad \mp i\frac{\pi q C\alpha'}{\theta}\epsilon^\mu{}_{\nu\rho\lambda}\partial(X^\nu + \alpha' k_1^\nu \ln|z|)\int dY^\rho \partial^\lambda G(X-Y)\Big|^2 \\
&\quad + \frac{1}{2\alpha'}\int dl\ |\dot{Y}-\alpha' k_2|^2 + \frac{qN}{\theta}(i \pm C)
\end{aligned}
$$

with topological linking number

$$N = \frac{\epsilon^{\mu\nu\rho\lambda}}{4\pi^2}\int d\Sigma_{\mu\nu}(X)\int dY_\rho \frac{(X-Y)_\lambda}{|X-Y|^4}$$

- By setting the squared terms to zero we minimize the action, and have only first-order differential equations

Lack of Instanton Solutions

- The equation for *Y* is trivial, and so is the solution:

$$Y(l) = \alpha' k_2 l,$$

The equation for *X*, however, is nontrivial,

$$z\partial X^\mu = \alpha' \left(\delta^\mu{}_\nu + i\frac{qC\alpha'}{4\theta} \epsilon^\mu{}_{\nu\rho\lambda} \frac{X_\perp^\rho \hat{k}_2^\lambda}{|X_\perp|^3} \right)^{-1} \left(\delta^\nu{}_\gamma - i\frac{qC\alpha'}{4\theta} \epsilon^\nu{}_{\gamma\kappa\sigma} \frac{X_\perp^\kappa \hat{k}_2^\sigma}{|X_\perp|^3} \right) k_1^\gamma .$$

but the solution is still trivial:

$$X = \alpha' k_1 \ln |z|$$

Hindsight is 20/20

- This makes perfect sense; the particle feels no force as $\theta g^2 \to \infty$, merely a statistical phase; there is nothing keeping the string open as another passes through it. This can be remedied by adding a *U(1)* coupling to the particle:

$$\Delta S_1 = \frac{1}{2\pi} \int d^2z \; J(z) A_\mu \bar{\partial} X^\mu$$
$$\approx iQ \int d\tau \; A_\mu \dot{X}^\mu$$

- The electrostatic repulsion will now keep it at a distance

$$R \sim \sqrt{g^2 q Q \alpha'}.$$

- Unfortunately this means we cannot take the same simplifying limit as before, since the force requires finite coupling.

Spin-Statistics Violations in Effective Field Theory

- Let us suppose that we <u>do</u> know the solutions, which will be of the form

$$X_N = \alpha' k_1 \ln|z| + f_N(|z|),$$
$$S_N = \frac{q}{\theta}\left(iN + C|N|\right).$$

- The amplitude including summation over linkings is

$$\mathcal{A}_{12} = \int d^2 z \sum_N e^{-k_2\cdot[\alpha' k_1 \ln|z| + f_N(|z|)] + iN/\theta - |N|C/\theta}$$

which corresponds to the effective string propagator

$$\Delta_{eff} = \frac{1}{2\pi}\int_0^\infty d\tau\ e^{-H\tau}\sum_N e^{F_N(H,\tau) + iN/\theta - |N|C/\theta}\int_{-\pi}^{\pi} d\sigma\ e^{i\sigma P}$$

Usual propagator keeps only $N=0$ contribution

Spin-Statistics Violations in Effective Field Theory

$$
\begin{aligned}
\Delta &= \int_0^\infty d\tau e^{-\tau H} \\
&= \frac{1}{H} = \frac{1}{p^2 - m^2} \\
\Delta_{eff} &= \int_0^\infty d\tau \sum_N e^{-\tau H + F_N(H,\tau) + iN/\theta - |N|C/\theta} \\
&\sim \int_0^\infty \frac{d\tau e^{-\tau H}}{1 - e^{-F(\tau,H)/\theta}} \\
&\sim \frac{1}{H^{1+\theta}} = \frac{1}{(p^2 - m^2)^{1+\theta}}
\end{aligned}
$$

This is precisely the nonlocal propagator that we wanted!

Experimental Constraints

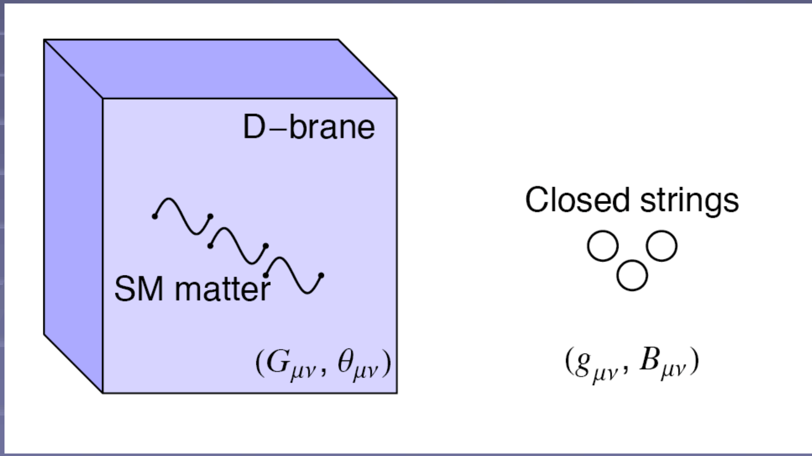
- Difficult to constrain this, since we don't know whether linkings scale with energy scale or some small parameter like θ.
- If it scales with energy, then for low string tension $\alpha' \sim (10\ \text{TeV})^{-2}$ we would see this at the LHC
- If it scales with a small parameter, low-energy but precise experiments such as VIP (Bartalucci et al. 2006) would see it due to the extraordinary bounds on the Pauli Exclusion Principle,

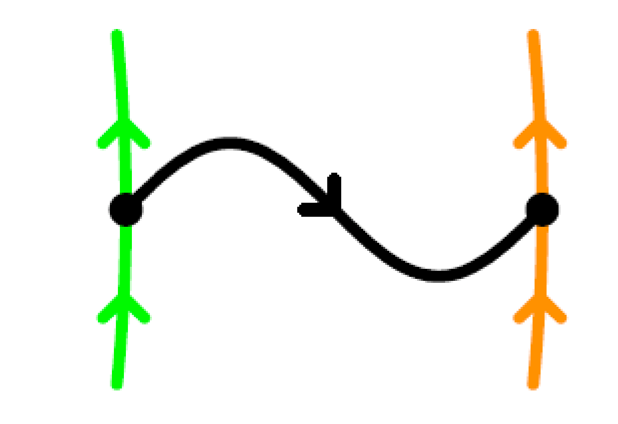
$$\frac{\beta^2}{2} \leq 4.5 \times 10^{-28}.$$

in terms of the Ignatiev-Kumzin-Greenberg-Mohapatra β parameter (Ignatiev and Kumzin 1987; Greenberg and Mohapatra 1989). This is even expected to improve 2 orders of magnitude in the next few years.

Method #2: Braneworlds

- Some string theory-motivated models of our universe imagine our 3+1 dimensions to be the worldvolume of a D-brane: (Blumenhagen, Cvetic, Langacker, Shiu 2005)





- Standard Model particles are open strings whose endpoints are attached to the brane with boundary conditions

$$g_{\mu\nu}(\partial - \bar{\partial})X^{\nu} + 2\pi\alpha' B_{\mu\nu}(\partial + \bar{\partial})X^{\nu}\Big|_{z=\bar{z}} = 0.$$

- This means the $g_{\mu\nu}$ and $B_{\mu\nu}$ fields get mixed together for open strings, and it is more natural to instead talk about the fields $G_{\mu\nu}$ and $\theta_{\mu\nu}$ which are each some combination of $g_{\mu\nu}$ and $B_{\mu\nu}$

Noncommutative Geometry

- These boundary conditions simplify considerably when taking a particular low-energy limit: (Seiberg and Witten 1999)

$$\theta^{\mu\nu} = (B^{-1})^{\mu\nu} \qquad \alpha' \sim \sqrt{\epsilon} \to 0 \qquad g_{\mu\nu} \sim \epsilon \to 0$$

- Then fields are multiplied according to the rule

$$\phi(x) \star \Phi(y) = e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}}\phi(x)\Phi(y).$$

which effectively means that coordinates don't commute by a constant,

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}.$$

- NC geometry is interesting and natural; if $[p,x] \neq 0$, why should $[x,y]=0$?

Nonlocality from Noncommutation

- To evade the Spin-Statistics Theorem there must be some sort of nonlocality, which can be detected in the spacelike-separated particle creation amplitude: (Chaichian, Nishijima, Tureanu 2002)

$$\langle 0|\,[:\phi(x)\star\phi(x):,:\phi(y)\star\phi(y):]|_{x^0=y^0}\,|p,p'\rangle$$
$$= -\frac{2i}{(2\pi)^{2d}}\frac{1}{\sqrt{\omega_p\omega_{p'}}}\left(e^{-ip'x-ipy}+e^{-ipx-ip'y}\right)\int\frac{d^3k}{\omega_k}\sin\left[k\cdot(x-y)\right]\cos\left(\frac{1}{2}k\cdot\theta\cdot p\right)\cos\left(\frac{1}{2}k\cdot\theta\cdot p'\right).$$

- This could only be nonzero if a timelike component of noncommutativity, θ^{i0}, is turned on.
- A totally timelike NC theory, $\theta^{\mu\nu}\,\theta_{\mu\nu} < 0$, yields inconsistent field theories (Gomis and Mehan 2000), but a totally lightlike NC $\theta^{\mu\nu}\,\theta_{\mu\nu} = 0$ is fine (Aharony, Gomis, Mehan 2000)

Why Doesn't Spatial NC Violate Spin-Statistics?

- It is surprising that simply turning on spatial NC doesn't produce some sort of spin-statistics violations, since it mixes up coordinates and thus destroys Poincaré symmetry
- A careful analysis of the generators shows that the Poincaré symmetry is still there but has simply been 'twisted'.

Spatial NC preserves SS

- Let's begin with a real scalar field ϕ,

$$\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left(a_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \right).$$

- This is multiplied using the expression given previously,

$$\begin{aligned}\phi(x) \star \phi(y) &= \int d^3k\ d^3p\ \tilde{\phi}(k)\tilde{\phi}(p) \left(e^{-ikx} \star e^{-ipy}\right) \\ &= \int d^3k\ d^3p\ \tilde{\phi}(k)\tilde{\phi}(p) e^{-ikx-ipy+\frac{1}{2}k\theta p}\end{aligned}$$

- But the occupation mode algebra is still unchanged,

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{p}}] = \delta_{\mathbf{k},\mathbf{p}}.$$

Spatial NC preserves SS

- It was claimed by Balachandran et al 2006 that the Fourier modes $\phi(k)$ themselves should also enter into this, since they furnish a representation of the Poincaré group:

$$P^i \tilde{\phi}(k) = k^i \tilde{\phi}(k).$$

$$\star \equiv e^{-\frac{i}{2}\theta^{\mu\nu} P_\mu P_\nu}$$

- Given the previous mode expansion, this means we should interpret the operators $a_{\mathbf{k}}, a^\dagger_{\mathbf{k}}$ as deformed relative to the usual ones $c_{\mathbf{k}}, c^\dagger_{\mathbf{k}},$

$$a_{\mathbf{k}} = c_{\mathbf{k}} e^{-\frac{i}{2} p_\mu \theta^{\mu\nu} P_\nu}, \qquad a^\dagger_{\mathbf{k}} = e^{\frac{i}{2} p_\mu \theta^{\mu\nu} P_\nu} c^\dagger_{\mathbf{k}}$$

Spatial NC preserves SS

- This would produce the modified commutation algebra

$$a_{\mathbf{k}}a_{\mathbf{p}} = e^{-ip\cdot\theta\cdot k}a_{\mathbf{p}}a_{\mathbf{k}}, \qquad a^\dagger_{\mathbf{k}}a^\dagger_{\mathbf{p}} = e^{-ip\cdot\theta\cdot k}a^\dagger_{\mathbf{p}}a^\dagger_{\mathbf{k}},$$
$$a_{\mathbf{k}}a^\dagger_{\mathbf{p}} = e^{ip\cdot\theta\cdot k}a^\dagger_{\mathbf{p}}a_{\mathbf{k}} + 2E_{\mathbf{k}}\delta^3(\mathbf{p}-\mathbf{k}).$$

  which would undo the usual (spatial) representation of the Moyal *-product, making field multiplication appear to be local, but which would now appear to modify spin-statistics!
- So <u>this suggests that we could interpret a noncommutative theory with usual spin-statistics as a commutative theory with modified spin-statistics</u>, which seems plausible given that space is now “mixed up”.

Spatial NC preserves SS

- **But this is not actually true (Tureanu 2006), once we recall that this means there must now be *three* Moyal NC multiplications,**

$$\phi(x) \star \phi(y) = \int d^3k\ d^3p\ \tilde{\phi}(k) \star e^{-ikx} \star \tilde{\phi}(p) \star e^{-ipy}.$$

trivial important trivial

- **The only one of these which is nontrivial is the middle one, which will produce exactly the same result as the original Moyal representaion**
- **<u>So purely spatial noncommutativity preserves the usual spin-statistics</u>**

Experimental Constraints

- There are bounds on spatial noncommutativity from several sources:
  - Lorentz violation (Kostelecky and Mewes 2002, 2003)
  - QCD gives $|\theta^{ij}|^2 < (10^{14}\text{ GeV})^{-2}$ (Mocioiu, Pospelov, Roiban 2000)
  - QED gives $|\theta^{ij}|^2 < (10\text{ TeV})^{-2}$ (Carroll, Harvey, Kostelecky, Lane, Okamoto 2001)
  - Constant B-flux would produce strange matter $\rho \sim \alpha^{-6}$ which must be rare (Nastase 2006)
- Some of these could be interpreted to place bounds on lightlike NC, the one of interest in SS violations (Kostelecky)
- But there may be difficulties in parameterizing this since by definition (Aharony, Gomis, Mehan 2000) $\theta^{\mu\nu}\theta_{\mu\nu} = 0$

Conclusion

- Does quantum gravity manifest itself as spin-statistics violations? Greenberg 2000 makes the interesting point that one cannot simply add statistics-violating terms to an action, maybe this is why we have found gravity difficult to quantize
- Can string theory produce such violations, and it is related to the Kalb-Ramond $B_{\mu\nu}$ field? Why don't we see such a field? Note that in 4d this is an axion since $dB = *da$.
- Is there any way to measure such a violation, and is it within reach of existing technology? How do the violations scale? (energy, small parameter, lightlike NC)

Thank you