Beyond Bose and Fermi Statistics

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Outline

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1. Introduction
2. Spin-statistics vrs. spin-locality
3. What quantum mechanics allows
4. Doplicher, Haag, Roberts analysis
5. Parastatistics
6. Quons
7. Anyons
8. Experimental tests of statistics

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Spin-statistics vrs. spin-locality

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Spin-statistics connection

1. *Particles* that have integer spin must obey Bose statistics.
2. *Particles* that have odd half-integer spin must obey Fermi statistics.
3. The spin-statistics connection concerns operators that create and destroy particles; i.e. asymptotic fields, which are free fields, considered by themselves.
4. When the connection is violated observables fail to be local, i.e. violate microcausality.

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Spin-locality connection

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What quantum mechanics allows

1. Messiah's "symmetrization postulate" (SP) that states of several identical particles are either symmetric or antisymmetric is equivalent to stating that identical particles only occur in one-dimensional representations of the symmtric group.
2. Concept of "ray" is replaced by "generalized ray."
3. In the general case one-body measurements cannot specify a state.
4. States in inequivalent representations of the symmetric group cannot intefere–superselection rule.
5. With some further conditions (CPT, Q, B, L conservation) no transitions between SP obeying and SP violating states can occur.

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How can you violate the usual statistics by a small amount?

1. The Hamiltonian (and all other observables) must commute with permutations of the identical particles; otherwise they would not be identical.
2. You can't just add a small violating term, $H = H_O + H_V$, where H_V doesn't commute with permutations.
3. You also can't introduce a new degree of freedom, like red electrons and blue electrons.
4. That would double the pair production cross section.
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Doplicher, Haag, Roberts analysis

1. Three types of statistics occur in 3 or more space dimensions.
2. Parabose statistics, integer order p.
3. Parafermi statistics, integer order p.
4. Infinite statistics.
5. In less than three space dimensions, fractional statistics (anyons) can occur.

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2. Bose case is the anticommutator, Fermi case is the commutator.
3. This leads to Green's trilinear commutation relations,
4. $[[a_k^\dagger, a_l]_\pm, a_m^\dagger]_- = 2\delta_{lm} a_k^\dagger$
5. To choose the Fock-like representation, Green added the usual vacuum condition, $a_k|0\rangle = 0$, and a condition on one-particle states $a_k a_l^\dagger|0\rangle = p\delta_{kl}|0\rangle$.

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Green's ansatz

1. Green found an infinite set of solutions of his commutation rules, one for each integer, with the ansatz $a_k^\dagger = \sum_{p=1}^n b_k^{(\alpha)\dagger}, \quad a_k = \sum_{p=1}^n b_k^{(\alpha)}$, where the $b_k^{(\alpha)}$ and $b_k^{(\beta)\dagger}$ are Bose (Fermi) operators for $\alpha = \beta$ but anticommute (commute) for $\alpha \neq \beta$ for the "parabose" ("parafermi") cases.
2. The index α is the "Green index."
3. The integer p is the order of the parastatistics.
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Physical interpretation

1. The physical interpretation of p is that, for parabosons, p is the maximum number of particles that can occupy an antisymmetric state, while for parafermions, p is the maximum number of particles that can occupy a symmetric state (in particular, the maximum number that can occupy the same state).
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Parastatistics as a quantum field theory

1. Norms of all states are positive, since sums of Bose and Fermi operator create states with positive norms.
2. Local observables, properly symmetrized, are analogous to the usual ones, for example, the local current for a spin-1/2 theory is $j^\mu = (1/2)[\bar{\psi}(x), \gamma^\mu \psi(x)]_-$.
3. Clustering properties hold because both commutators and anticommutators decrease exponentially (for massive fields) for large spacelike separation.
4. The fields transform under the Poincaré group in the usual way.
5. The spin-statistics connection holds.
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Parastatistics as an interpolation between Bose and Fermi theory

1. For $p = 1$ parabosons are Bosons. For $p \geq 2$, p particles can occur in an antisymmetric state.
2. For $p = 1$ parafermions are Fermions. For $p \geq 2$, p particles can occur in a symmetric state.
3. The violations of statistics provided by parastatistics are gross. Parafermi statistics of order 2 has up to 2 particles in each quantum state. High-precision experiments are not necessary to rule this out for all particles we think are fermions.
4. The analogous statement holds for parabosons. Thus parastatistics is not useful to describe small violations of statistics.

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Relation between parastatistics, order p, and $SU(p)$ or $SO(p)$ theories

1. Can relate parastatistics, order p, to a theory with a p-valued hidden degree of freedom.
2. Use a Klein transformation to convert the Green component fields with the “wrong” relative commutation relations to the “normal” ones.
3. Ohnuki and Kamefuchi and Drühl, Haag and Roberts each studied this.
4. With only baryon number zero observables, such as $\bar{\psi}\gamma^\mu\psi$, one gets $SU(p)$.
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Apparent Violations of Statistics Due to Compositeness

1. Consider two identical Fermi nuclei at locations A and B. Assume they have the same polarization.
2. In close proximity the exclusion principle may force each of the nuclei into excited states with small amplitudes $\lambda_A \neq \lambda_B$.
3. Let the creation operator for the nucleus at location A be $N^\dagger_A = \sqrt{1-\lambda^2_A} b^\dagger_0 + \lambda_A b^\dagger_1 + \cdots, |\lambda_A| << 1$ with the analogous expression for the nucleus at B.
4. The creation operators obey $[b^\dagger_i, b^\dagger_j]_+ = 0$.
5. Then $b^\dagger_A b^\dagger_B|0\rangle = [\sqrt{1-\lambda^2_A}\lambda_B - \lambda_A\sqrt{1-\lambda^2_B}] b^\dagger_0 b^\dagger_1|0\rangle$, $\|b^\dagger_A b^\dagger_B|0\rangle\|^2 \approx (\lambda_A - \lambda_B)^2 << 1$, so, with small probability, the two could even occupy the same location, because each could be excited into higher states with different amplitudes.
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Apparent Violations of Statistics Due to Compositeness

1. Consider two identical Fermi nuclei at locations A and B. Assume they have the same polarization.
2. In close proximity the exclusion principle may force each of the nuclei into excited states with small amplitudes $\lambda_A \neq \lambda_B$.
3. Let the creation operator for the nucleus at location A be $N_A^\dagger = \sqrt{1-\lambda_A^2}b_0^\dagger + \lambda_A b_1^\dagger + \cdots$, $|\lambda_A| << 1$ with the analogous expression for the nucleus at B.
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Infinite statistics

1. Roger Hegstrom suggested to average the Bose and Fermi commutation relations.
2. $(1/2)\{[a_k, a_l^\dagger]_- + [a_k, a_l^\dagger]_+\} = a_k a_l^\dagger = \delta_{kl}$.
3. Cuntz algebra.
4. With the Fock-like vacuum condition, $a_k|0\rangle = 0$, can calculate all matrix elements.
5. No condition on two a's or two $a^\dagger$'s.
6. The norm of every monomial in $a^\dagger$'s acting on the vacuum is one.
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Physical significance of quons

1. All representations of the symmetric group on n objects occur.
2. For $q \to -1$ the more antisymmetric representations dominate.
3. For $q \to 1$ the more symmetric representations dominate.
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Speicher's probabilistic ansatz

1. To get the Fock-like representation of the quon algebra, Speicher chose a probabilistic condition for the signs $s^{(\alpha,\beta)}$, $\mathrm{prob}(s^{(\alpha,\beta)} = 1) = (1+q)/2$, $\mathrm{prob}(s^{(\alpha,\beta)} = -1) = (1-q)/2$.
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2. Composites of even (odd) numbers of fermions are bosons (fermions).
3. For quons, $q_{composite} = q_{constituent}^{n^2}$.
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1. The energy of widely separated sub systems must be additive.
2. This requires $[\mathcal{H}(x), \phi(y)]_- \to 0, \ |x-y| \to \infty$.
3. For Fermi fields, this requires an even number of Fermi fields in observables.
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How the space dimension enters–angular momentum

1. In one dimension there is no spin, since there is no axis to rotate about.
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1. For two identical particles in 2 space dimensions, go to center of mass and relative coordinates.
2. Assume we remove coincident points and identify the points that correspond to transposition of the particles.
3. This is irrelevant for the center of mass coordinate.
4. For the relative coordinate we get the plane with the origin removed and antipodal points identified.
5. We move a point around a closed path in this space.
6. If the path can be contracted to the starting point, no phase can occur from this motion.
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How the space dimension enters–more than one space dimension–the representations of the rotation group, $O(D)$.

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2. At least for simple groups we can reduce ray representations to true representations by going to the universal covering group, whose group space is simply connected, i.e., in which all paths are contractible.
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Thank you

In conclusion, I thank Professor Milotti and the other organizers for arranging this workshop and for asking me to speak here.