Spin-statistics transmutation in Quantum Field Theory

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J. Froehlich, P.A. M., Lett. Math. Phys. 16 (1988) 347 , Commun. Math. Phys. 121 (1989) (anyons)

K. Lechner, P.A. M., JHEP 0012 (2000) 028 (dyons)

P.A. M.,Czech. J. Phys. 52 (2002) C461

What is spin-statistics transmutation

- Spin-statistics transmutation, borrowing a terminology used in planar systems (Polyakov 1988), is the phenomenon occurring when a "dressing" interaction modifies the "bare" spin and statistics of particles or fields.
- Historically it first appeared in Quantum Mechanics (QM) and Semiclassical Quantum Field Theory (QFT) settings
- Here we sketch how to implement such phenomenon in fully quantized (relativistic) field theory using euclidean correlation functions (correlators).

Plan of the talk

- Historical remarks on spin-statistics transmutation in QM and semiclassical QFT
- The problem of the extension to operators or correlators in fully-quantized QFT (beyond semiclassical approximation)
- The solution: Dirac ansatz for gauge-invariant fields and how it opens the way to spin-statistics transmutation in QFT
- Application to anyons in 2+1 D
- Application to dyons in 3+1 D

Transmutation in QM (Historical remarks)

- Probably the first considered example of transmutation of spin (from half-integer to integer, implicit in Tamm 1931) was for dyons: composites of a magnetic monopole and a charged particle (-> spin1/2 electron)
- Classically , splitting the location of the electric charge *e* of the particle and the magnetic charge *g* of the monopole by a distance **a** ≠0 along the 3–axis, the angular momentum **J** stored in the generated electromagnetic field $\mathbf{E} = e/4\pi\, (\mathbf{x}-\mathbf{a})|\mathbf{x}-\mathbf{a}|^3$, $\mathbf{B}= g/4\pi\, \mathbf{x}|\mathbf{x}|^3$ is given by

  $J_3 = \int d^3x[\mathbf{x} \wedge (\mathbf{E} \wedge \mathbf{B})]_3 = eg/4\pi.$

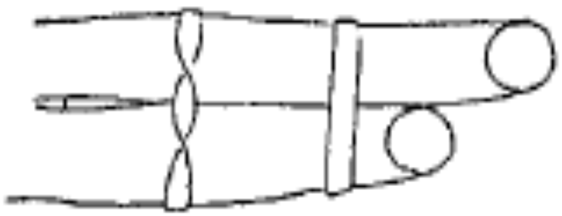
Transmutation in QM: dyons

- QM requirement: spectrum of $J_3 = eg/4\pi \in \mathbf{Z}/2$ ($\hbar = 1$)
-> Dirac (1931) quantization condition $eg \in 2\pi\mathbf{Z}$.
If $eg/2\pi$ is odd and the charged particle is a boson/fermion (-> integral/half-integral "bare" spin) the classical calculation suggests that the composite dyon carries half–integral/integral spin (spin transmutation)
- Rigorously proved in QM (Hurst 1968) ; later on shown to survive in QFT with a semiclassical treatment of monopole in Yang-Mills theories (Jackiw-Rebbi, Hasenfratz-t' Hooft 1976)
- Defining a physical (gauge-invariant) wave function for the dyon composites Goldhaber (1976) showed that the usual spin-statistics connection holds for them.

2+1 D and the rubber band lemma

- Similar phenomena (Wilczek 1982) occur in planar systems for charge-magnetic flux (vortex) composites. [assume here spin 0 for the charged particle]
- For semiclassical vortex -> geometrical interpretation of the spin-statistics connection (Wilczek-Zee 1983) inspired by “rubber-band lemma” (Rubinstein-Finkelstein 1968): if a rubber band is wrapped twice about a rod, it can exhibit a self-crossing together with a 2π twist (-> are topologically equivalent)

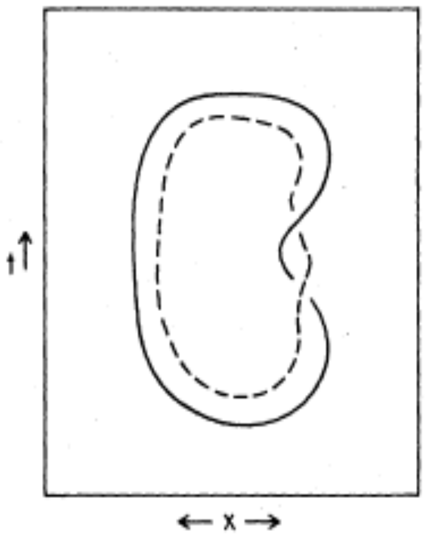
self-crossing



2π twist

Spin as twist, exchange as self-crossing

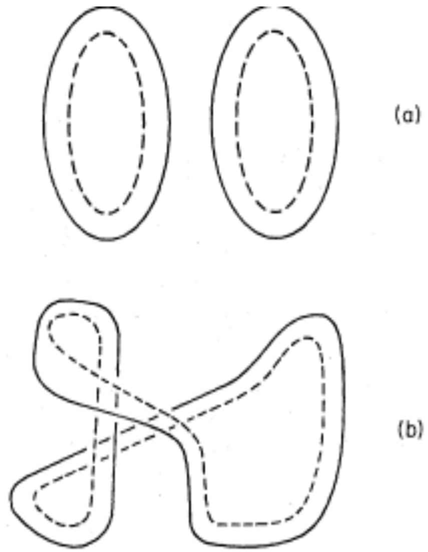
- Imagine the two boundaries of the rubber band describe worldlines of charge (*e*) and vortex- magnetic flux (Φ), shifted by UV regulator **a** ≠0 as for dyons.
- The 2π twist describes a 2π rotation ->spin, the self-crossing an exchange with orientation (under- and over-crossing are distinct in 2+1 D) ->statistics.



<-2π rotation exchange ->

continuous line=electric flux

dashed line= magnetic flux



Transmutation in QM: anyons

- The phase factor produced by self-crossing ≈ 2π twist can computed by Aharonov-Bohm effect (charged particle in magnetic flux). It is given by 2π $e\Phi$ times the linking number # of electric and magnetic flux lines involved (#=± 1)

- -> $e\Phi$ mod **Z**=spin and statistics for such composites (->transmutation) in a semiclassical treatment of the vortex. In 2+1 D both the spin (S->Irre-pr-rep of SO(2)) and the statistics (θ-> Irre-pr-rep of braid group generated by oriented exchanges) can be labelled by ANY number ∈ [0,1[: these composites were called ANYons .

Problems in extension to QFT

- In QM and semiclassical treatment only **closed** worldlines of particles appear (particles cannot be created/annihilated): crucial for deriving the above topological results for spin and statistics .
- In fully-quantized QFT (Feynman-Schwinger-) representation of field correlators in terms of the worldlines of particles exhibits also **open paths** with ends corresponding to the insertion of charged fields, where particles are created/annihilated.
- How to extend the topological arguments of QM to QFT with open paths, where topological stability disappears?
  Way out: Dirac ansatz for non-local fields.

Transmutation in QFT: non-local fields

- In gauge theories one cannot construct a **local** charged field operator acting on a physical (positive-metric) Hilbert space of states (Strocchi 1977)
- Example (trivial for transmutation, but easy and familiar): Quantum Electro Dynamics (QED) in the operator approach. Let $|\Omega>$ =physical vacuum $\hat{\psi}(\mathbf{x})$= **local** electron field operator, $\mathbf{x} \in \mathbf{R}^3$
- $\hat{\psi}(\mathbf{x}) |\Omega>$ is not a state in the physical Hilbert space (even with an U.V. regulator). Let Q_{BRS} be the charge selecting the space of physical states, $|$ phys $>$, by $Q_{BRS}|phys> = 0$, we have

$$[Q_{BRS} , \hat{\psi}(\mathbf{x})] \neq 0 \to Q_{BRS}\, \hat{\psi}(\mathbf{x}) |\Omega> \neq 0 \quad (1)$$

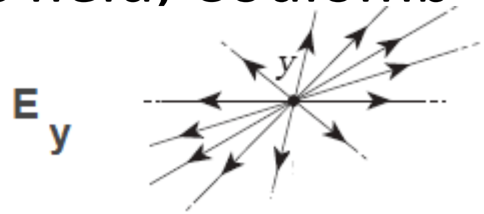
- (However perturbatively $[Q_{BRS} , \hat{\psi}^{as}(\mathbf{x})] = 0$)

Dirac ansatz

- Basic motivation of (1): $\hat{\psi}(\mathbf{x})$ is not gauge invariant.
- To turn it into a gauge-invariant field operator Dirac (1955) proposed the following ansatz . Let $\hat{\mathbf{A}}$ denote the quantum photon gauge field and $\mathbf{E}_\mathbf{x}$ a classical electric field, Coulomb-like, satisfying

$$\text{div}\, \mathbf{E}_\mathbf{x} = \delta_\mathbf{x} \qquad (2)$$

(1 D suppressed)



- Then a "physical electron operator" is formally given by

$$\hat{\psi}(\mathbf{x}) \exp[i \int d^3\mathbf{y}\, \hat{\mathbf{A}}(\mathbf{y}) \cdot \mathbf{E}_\mathbf{x}(\mathbf{y})],$$

it is gauge-invariant, due to (2) but non-local.
[gauge-invariance: $\hat{\psi}(\mathbf{x}) \to \hat{\psi}(\mathbf{x})\, e^{i\Lambda(\mathbf{x})}$, $\hat{\mathbf{A}}(\mathbf{y}) \to \hat{\mathbf{A}}(\mathbf{y}) + \boldsymbol{\partial}\Lambda(\mathbf{y})$,
(2)-> $\int d^3\mathbf{y}\, \boldsymbol{\partial}\Lambda(\mathbf{y}) \cdot \mathbf{E}_\mathbf{x}(\mathbf{y}) = - \int d^3\mathbf{y}\, \Lambda(\mathbf{y})\, \text{div}\, \mathbf{E}_\mathbf{x}(\mathbf{y}) = - \Lambda(\mathbf{x})$]
- The **E-**dependent phase describes the Coulomb photon cloud tied to the electron even asymptotically.

Dirac ansatz and Euclidean QFT

- Euclidean correlators of the field operators $\hat{\psi}(\mathbf{x})\ \exp[i \int d^3\mathbf{y}\ \hat{\mathbf{A}}(\mathbf{y})\cdot \mathbf{E}_\mathbf{x}(\mathbf{y})]$ (heuristically) have the following form

  $<\ldots.\ \psi(x)\ \exp[i \int d^4y\ A(y)\cdot E_x(y)]\ldots>$

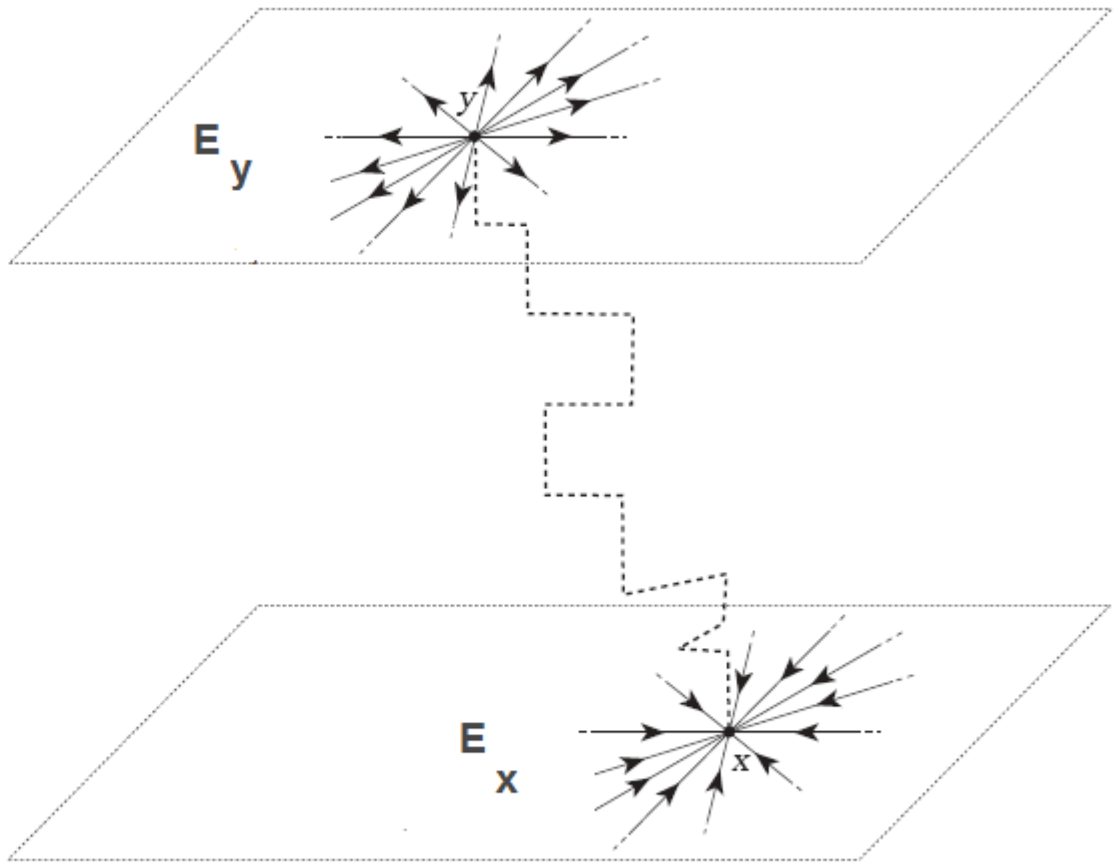
  where $x = (x^0, \mathbf{x}) \in \mathbf{R}^4$, $\psi(x)$ is a Grassmann field, $A(x)$ the gauge field, E_x is an electric current distribution related to $\mathbf{E}_\mathbf{x}$ by $E_x(y) = (0, \mathbf{E}_\mathbf{x}(y)\ \delta(y^0 - x^0))$ [so that $\mathrm{div}\ E_x = \delta_x$] and <…> denotes the average in the euclidean path-integral measure for QED .
- An OS-like reconstruction theorem allows to reconstruct from these Green functions (with UV cutoff) the corresponding non-local field operators.

Worldline representation

- Integrating out ψ in euclidean QED one can obtain a Feynman-Schwinger-like representation of correlators in terms of world lines of two kinds:
- closed, corresponding to virtual particle-antiparticle pairs, and open with boundary on the points of "physical" field insertions, corresponding to creation and annihilation of electrons/positrons.
- At these boundary points the electric flux flowing through the open worldlines is spread out through the electric current distributions E , thus preserving current conservation (-> gauge invariance) in spite of particle creation/annihilation.

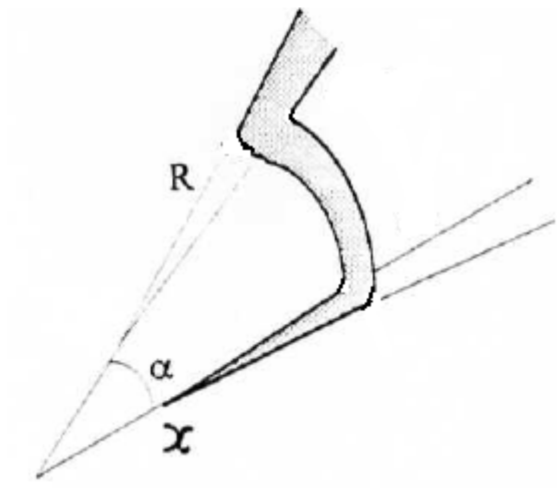
Electric current distributions and open electron worldline in a 2-point euclidean correlator of physical electron field

(1 D suppressed)



Rotations for Dirac ansatz

- The "physical" charged fields are non-local, with a tail reaching infinity, hence a rotation by an angle α should be defined as limit R →∞ of a IR cutoff rotation $U(\alpha)^R$, acting as a rotation by α within a ball of radius R, smoothly interpolating to the identity between R and R + 1 and acting trivially outside a ball of radius R + 1 (-> rotation generated by a local current).



Flux lines of $U(\alpha)^R$ (E_x)

- 2π -rotation leaves invariant all local observables -> (Schur's lemma) its action on charged states is represented by a phase factor, $e^{i2\pi S}$, where S identifies the spin (better the spin-type, i.e. the spin modulo **Z**).

Spin and statistics for Dirac ansatz

- Euclidean :

$\lim_{R\to\infty}$ <... $U(2\pi)^R$ ($\psi(x)$ exp[$i \int d^4y\, A(y)\cdot E_x(y)$])...>

= $e^{i2\pi S}$<... $\psi(x)$ exp[$i \int d^4y\, A(y)\cdot E_x(y)$]... >

A priori S has two contributions: local, from the local field $\psi(x)$ (-> "bare" spin), and topological at infinity (trivial in QED), from the dressing, due to the rotated E_x, which may transmute the spin.

- Analogously an exchange σ on non-local fields should be defined as lim $R\to\infty$ of an IR cutoff exchange $U(\sigma)^R$ In the limit it yields (instead of $e^{i2\pi S}$) the statistics factor $e^{i2\pi\theta}$, with a possible contribution from ∞ (trivial in QED), due to the exchanged Dirac dressings.

Spin-statistics transmutation in QFT

- The (topological) contribution at infinity of the dressing factor of the Dirac ansatz under 2π– rotation and exchange opens the way to spin-statistics transmutation in fully-quantized QFT because it can give an additional contribution to the spin and statistics phase factors of the local (non-gauge invariant) field .
- *Examples: anyons, dyons, skyrmions (through a non-abelian version of the dual of the Dirac ansatz,* [*J. Froehlich, P.A. M., Nucl. Phys. B 335 (1990) 1*] *).*

Anyons: QFT model

- In real physics anyons appear as fractionally charged carriers in the Fractional Quantum Hall Effect , exhibited by a 2D electron gas in presence of impurities in a strong magnetic field at low temperature.
- QFT model example: theory with a complex scalar (spin 0) field ϕ and an abelian gauge field, A, with action

$$S(A, \phi)=\int \tfrac{1}{2}|(\partial_\mu - A_\mu)\, \phi|^2 + \tfrac{1}{2} m^2|\phi|^2 + k/4\pi i\, \varepsilon^{\mu\nu\rho} A_\nu \partial_\mu A_\rho\ ,$$

the last term is the Chern-Simons action

(Model dual to that for the FQHE, in a relativistic version)

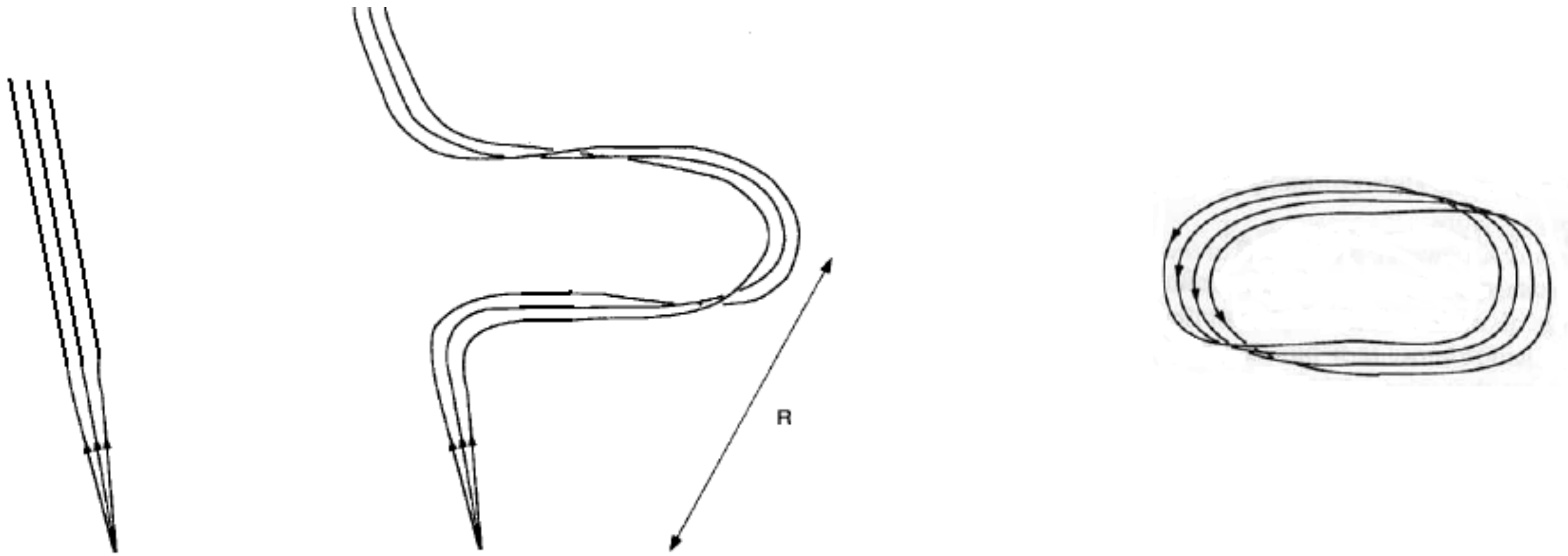
- We discuss here the euclidean approach, because easier to render mathematically rigorous (with UV cutoff) .

Anyons: non-local physical field

- E_x, $x \in \mathbf{R}^3$ = 3-d Coulomb-like electric field satisfying div $E_x = \delta_x$, with support in a cone in the positive (negative) time half-space if $x^0 \geq 0$ ($x^0 \leq 0$) to have positive metric in the OS reconstructed Hilbert space of states (modified euclidean Dirac ansatz).
- Gauge-invariant "physical" euclidean anyon field $\phi(E_x) = \phi(x) \exp[i \int d^3y\, A(y) \cdot E_x(y)]$
- Via OS one can reconstruct anyon field operators $\hat{\phi}(\mathbf{E_x})$ with support on wedges (2D Buchholz-Fredenhagen)
- $\phi(x)$ carries electric charge 1 (by minimal coupling to A) and magnetic flux 1/2k (by Chern-Simons) . Therefore along E_x flow both an electric and a magnetic flux; with a UV regulator we split the support of the two fluxes.

Anyons: spin transmutation

- An IR cutoff 2 π rotation acts on the dressing factor in the anyon field $\phi(E_x)$ producing a phase factor proportional, as R ->∞ , to the linking number of electric and magnetic flux lines. In formulas:
- Define α_μ^R by $U(2\pi)^R(E^\mu_x) - E^\mu_x = \varepsilon^{\mu\nu\rho}\partial_\nu \alpha_\rho^R = {}^*\partial\alpha^{R\mu}$



E_x $U(2\pi)^R(E_x)$ ${}^*\partial\alpha^R$

Anyons: spin transmutation- computation

$\lim_{R\to\infty}$ <.... $U(2\pi)^R$ ($\phi(x)$ exp[$i\int d^3y\, A(y)\cdot E_x(y)$]))...>=

$\lim_{R\to\infty}$ <.... $\phi(x)$ exp[$i\int d^3y\, A(y)\cdot (E_x(y)+ {*\partial\alpha^R}(y)$]...>=

$\lim_{R\to\infty}$ exp[$i\,\pi/k \int \varepsilon^{\mu\nu\rho}\,\alpha_\mu{}^R\,\partial_\nu\,\alpha_\rho{}^R + O(1/R)$]

<.... $\phi(x)$ exp[$i\int d^3y\, A(y)\cdot E_x(y)$]))...>=

exp[$i\, 2\pi/2k$] <.... $\phi(x)$ exp[$i\int d^3y\, A(y)\cdot E_x(y)$]))...>

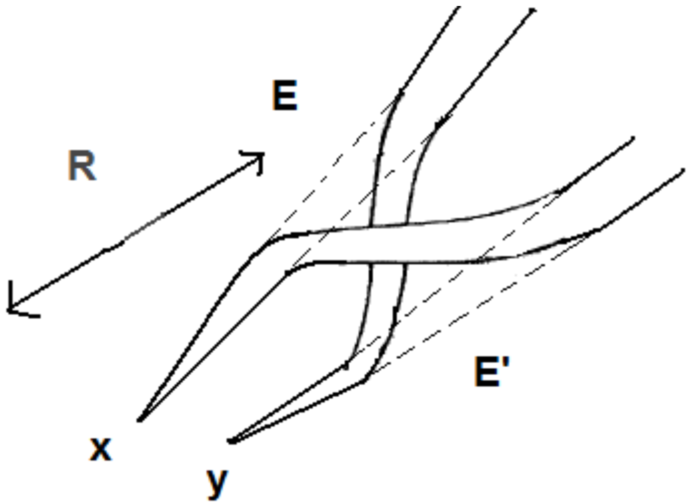
- $\int \varepsilon^{\mu\nu\rho}\,\alpha_\mu{}^R\,\partial_\nu\,\alpha_\rho{}^R$ = linking number # of the electric and magnetic flux lines of $U(2\pi)^R(E^\mu{}_x)$- $E^\mu{}_x$ (#=1)
- The term O(1/R) comes from the self-interaction of $U(2\pi)^R(E^\mu{}_x)$- $E^\mu{}_x$
- Hence, although $\phi(x)$ has spin 0, the physical anyon field has spin type S = 1/(2k). (spin transmutation)

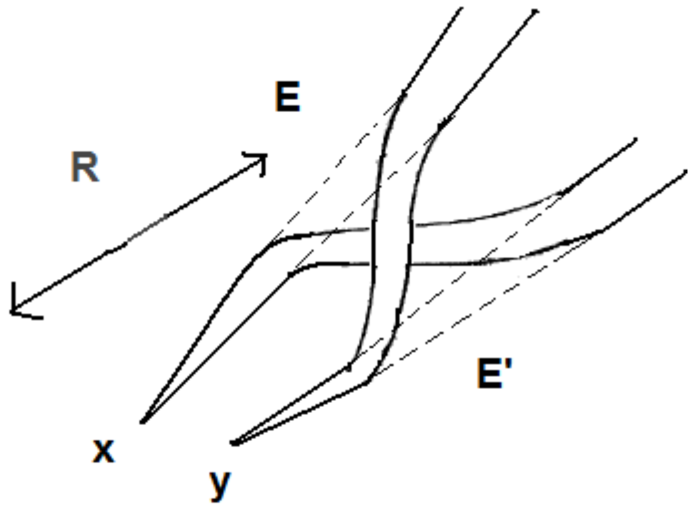
Anyons: statistics transmutation

- Analogously , consider an IR cutoff exchange with orientation $U(\pm\sigma)^R$ acting on the product of two fields $\phi(E_x)$, $\phi(E'_y)$ with non-overlapping supports.

- As $R \to \infty$ it yields a phase factor proportional to the linking number ($\int \varepsilon^{\mu\nu\rho} \alpha_{\pm\mu}^{R} \partial_{\nu} \alpha_{\pm\rho}^{R} = \pm 1$) of electric and magnetic flux lines of $U(\pm\sigma)^R (E^{\mu}_{x} + E'^{\mu}_{y})$- $(E^{\mu}_{x} + E'^{\mu}_{y})) = \varepsilon^{\mu\nu\rho} \partial_{\nu} \alpha_{\pm\rho}^{R}$ producing a statistics transmutation to a statistics parameter $\theta = \pm 1/(2k)$.

Spin-statistics connection for anyons





$U(+\sigma)^R(E'_y+E_x)$ $\quad\quad$ $U(-\sigma)^R\,(E_x+E'_y)$

$$\hat{\phi}(\mathbf{E_x})\hat{\phi}(\mathbf{E'_y}) = \lim_{R\to\infty} U(+\sigma)^R\,\hat{\phi}(\mathbf{E'_y})\hat{\phi}(\mathbf{E_x}) = \exp[i\,\pi/k]\,\hat{\phi}(\mathbf{E'_y})\hat{\phi}(\mathbf{E_x})$$

- Spin statistics connection : $S=\theta=1/2k$
- Follows simply from the rubber band lemma applied to the “rubber band” of electric and magnetic flux pushed at infinity in the Dirac dressing

Dyons: QFT model

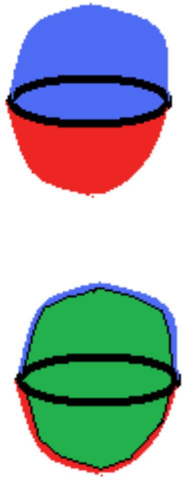
- In real physics dyons should appear in Grand-Unified and in Supersymmetric (e.g. Seiberg-Witten like) Yang- Mills Theories. The dynamic of dyons is described by Dirac-Maxwell equations (Dirac 1948): $\partial^{\mu}F_{\mu\nu}=ej_{\nu}$ $\partial^{\mu}\tilde{F}_{\mu\nu}=gj_{\nu}$ where j_{ν} is the dyon current and $\tilde{F}_{\mu\nu}=\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$
- The coupling of the dyon-current with the gauge fields is given by $i \int j^{\mu} (e A_{\mu}+g \tilde{A}_{\mu})$.
- As in QM setting , the QFT is consistent only if the Dirac quantization condition $eg \in 2\pi\mathbf{Z}$ is satisfied.

Dirac surfaces

- In fact, the magnetic poles carried by dyons are attached to Dirac strings, sweeping in their time evolution 2-surfaces Σ (Dirac surfaces), which physically should be unobservable.
- In the effective action, obtained integrating out the gauge fields in the partition function, the Dirac surfaces appear (Schwinger 1966) in the term
  $i\, eg \int j^{\nu} \Delta^{-1} \partial^{\mu} \Sigma_{\mu\nu}$ (Δ=4D Laplacian).
  where $\Sigma_{\mu\nu}$ is the surface current corresponding (Poincare` dual) to the Dirac surface Σ whose boundary is the support of j.

Dirac strings invisible in effective action

- A change of the Dirac surface from Σ to a new surface Σ’ with the same boundary can be realized by shifting $\Sigma_{\mu\nu}$ by $\partial_{[\mu} V_{\nu]}$ with V_μ a volume current corresponding to the volume V bounded by Σ’- Σ.



- In the effective action Σ→Σ’ produces a term $i\, eg \int j^\nu \Delta^{-1} \partial^\mu \partial_{[\mu} V_{\nu]} = i\, eg \int j^\mu V_\mu \in i2\pi\mathbf{Z}$ if $eg \in 2\pi\mathbf{Z}$, since j^μ and V_μ are **Z**-valued; hence the Dirac string is invisible, as physically required.

Problem of Dirac ansatz with E_x

- If the currents j^{ν} are associated (via Feynman-Schwinger) to a "bare" scalar dyon field $\phi(x)$, the physical dyon field constructed according to Dirac ansatz would be
  $\phi(x) \exp[i \int d^4y\, (e\, A + g\, \tilde{A})(y) \cdot E_x\, (y)]$,
  with E_x the 3D electric Coulomb distribution field
- In correlators of physical dyon field the integral current j is shifted by the non-integral current E_x. A change of Dirac surface $\Sigma \rightarrow \Sigma'$ produces now an additional term in the effective action:
  $ieg \int E_x^{\ \mu} V_{\mu}$ not in $i2\pi\mathbf{Z}$ even if $eg \in 2\pi\mathbf{Z}$,
  since E_x is not integer. Hence the Dirac string, unphysically, becomes visible...

Problem with Mandelstam string

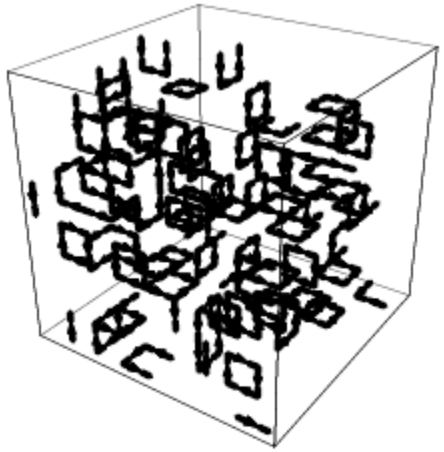
- To recover for dyon correlators the independence on the Dirac string we need to substitute E_x by an integer current j_x [div $j_x = \delta_x$] with support on a path at fixed time starting from x and reaching infinity (Mandelstam (1962) string). Since j_x is integral $i\, eg \int j_x^{\mu} V_{\mu} \in i2\pi\mathbf{Z}$ if $eg \in 2\pi\mathbf{Z}$ and the Dirac string is now invisible again.

- This choice, however, produces IR divergences due to the ∞ self-energy of the string (currents do not decay sufficiently fast at infinity).

Fluctuating Mandelstam strings

- To avoid IR divergences, one has to replace a fixed Mandelstam–string j_x by a sum over fluctuating Mandelstam–strings, weighted by an appropriate measure $D\mu(j_x)$, supported on strings which fluctuate so strongly that the interaction energy between two strings is finite, even for an infinite length.
- This measure $D\mu(j_x)$, with UV lattice cutoff, exists (Froehlich-M. 1999) and at large distances
  $\int D\mu(j_x)\exp[i\int j_x \cdot A] \approx \exp[i\int E_x \cdot A]$
  Hence, on large scales the fluctuating Mandelstam strings produce a phase factor with the same safe infrared behaviour of the (standard) Dirac ansatz

Dyons: modified Dirac ansatz



- The result on IR behaviour was checked by numerical lattice computation (Belavin-Chernodub-Polikarpov 2001)
  Simulation of a typical configuration of a string j_x of $D\mu(j_x)$
- Therefore the right "physical dyon field" is $\phi(x) \int D\mu(j_x) \exp[i \int d^4y \, (e\, A + g\, \tilde{A})(y) \cdot j_x(y)]$ and it can be shown formally to satisfy the requirements for OS reconstruction -> one can obtain from its correlators a dyon field operator.
- This kind of structure of the physical field is unexpected on the basis of a semiclassical treatment! However it is (to my knowledge) the only consistent when dynamical charges and monopoles coexist.

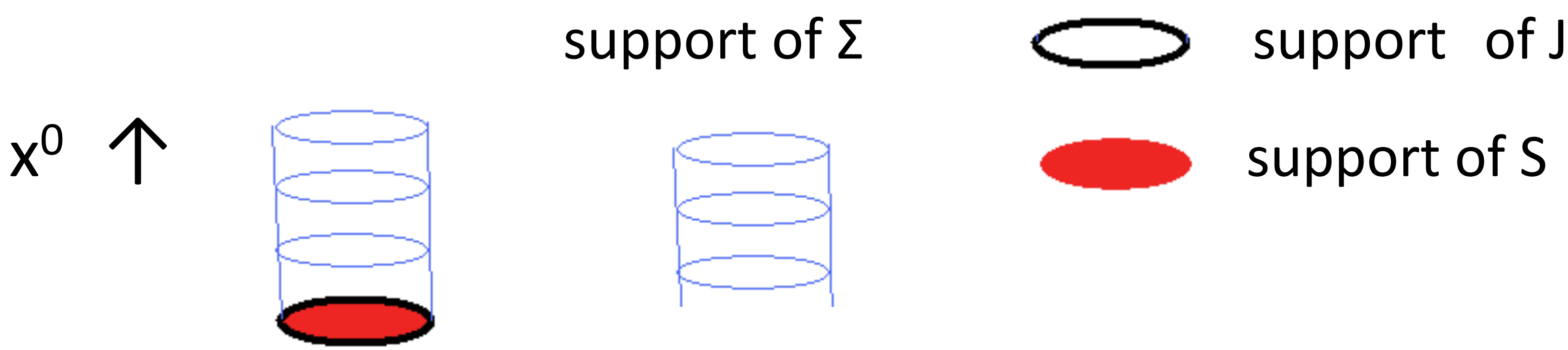
Linking number for dyons

- Perhaps unexpectedly, using the above defined dyon field one can export to dyons in 3+1 D, where no obvious concept of linking exists, the spin-statistics consideration of linking numbers in 2+1 D discussed before.
- The IR cutoff rotation $U(2\pi)^R$ acts deforming j_x producing a contribution to the effective action $i\, eg \int j^{R\,\nu} \Delta^{-1} \partial^{\mu} \Sigma^{R}{}_{\mu\nu}$ where j^R is the current corresponding to $U(2\pi)^R (j_x) - j_x$ and Σ^R is the surface current corresponding to a surface bounded by the support of j^R.

Linking: choice of Dirac surface

- Assume j^{R} at (euclidean) time 0. Choose Σ^{R} directed upward in time:

  $\Sigma^{R}_{ij}(x^0, \mathbf{x}) = H(x^0)\, j^{Rk}(\mathbf{x})\epsilon_{ijk}$ and let S^{R}_{kl} denote the surface current corresponding to the surface at constant 0-time bounded by the support of j^{R},
  i.e. $j^{R}_{k}(\mathbf{x}) = \partial^{l} S^{R}_{kl}(\mathbf{x})$.



Dyons: spin - computation

- $ieg\int d^4x\, j^{R\,\nu}(x) \int d^4y\, \Delta^{-1}(x\text{-}y)\, \partial^\mu \Sigma^R{}_{\mu\,\nu}(y) =$

$ieg\int d^3\mathbf{x}\, j^{R\,j}(\mathbf{x}) \int d^3\mathbf{y} \int dx^0 H(x^0)\, \Delta^{-1}(x^0,\mathbf{x}\text{-}\mathbf{y})\, \varepsilon_{ijk}\, \partial^i j^{Rk}(\mathbf{y}) =$

$ieg\int d^3\mathbf{x}\, j^{R\,j}(\mathbf{x}) \int d^3\mathbf{y}\, ½\, \Delta_3{}^{-1}(\mathbf{x}\text{-}\mathbf{y})\, \varepsilon_{ijk}\, \partial^i j^{Rk}(\mathbf{y}) =$

$½ieg\int d^3\mathbf{x}\, \partial_m S^{Rmj}(\mathbf{x}) \int d^3\mathbf{y}\, ½\, \Delta_3{}^{-1}(\mathbf{x}\text{-}\mathbf{y})\, \varepsilon_{ijk}\, \partial^i \partial_l S^{Rkl}(\mathbf{y}) =$

$½ieg\int d^3\mathbf{x}\, \partial_m S^{Rmj}(\mathbf{x})\, \varepsilon_{jkl}\, S^{Rkl}(\mathbf{x})$

where Δ_3 is the 3D laplacian.

The integral after UV regulation gives the same linking number computed for the spin of the anyon ($\alpha^R \to S^R$) . Hence the result is independent of the Mandelstam string j_x in $D\mu(j_x)$.

Dyons: spin-statistics transmutation

- Result : spin of the physical dyon field S
 satisfies $\exp[i\, 2\pi\, S]=\exp[\frac{1}{2} i e g]$ and if $eg= 2\pi\, n$ with n odd we have spin transmutation.
Analogously, using the IR cutoff exchange one can prove also the statistics transmutation in dyon QFT.
- Thus we have shown that choosing the Dirac–strings along the time direction both spin and statistics of the dyon field are related to the linking numbers of electric and magnetic fluxes appearing in a deformation of Mandelstam strings in a three–dimensional space at fixed time and again spin-statistics connection follows from the “rubber band” lemma.

Summary: spin-statistics transmutation in QFT

- **In QFT with local gauge invariance, a local non-gauge invariant quantum field has "bare" spin and statistics which might be modified by the dressing transformation necessary to render the field gauge-invariant.**
- **This dressing can be obtained by a suitable version of Dirac ansatz and the resulting "physical" field is non-local with a tail reaching infinity.**

Summary: spin-statistics transmutation in QFT

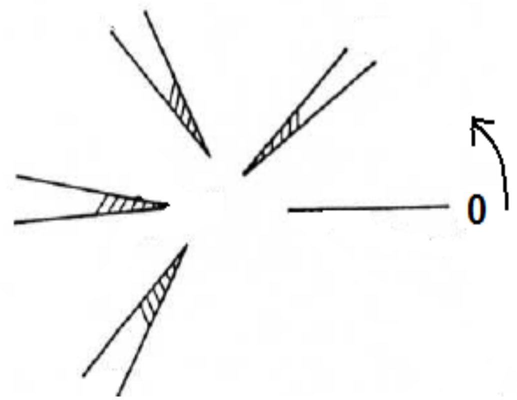
- **One computes the spin (statistics) of the non-local physical fields by applying an IR cutoff 2π-rotation (an IR cutoff exchange) and removing the IR cutoff by a limiting procedure.**
- **Topological contributions at infinity of the Dirac dressing induce the spin/statistics transmutation and spin-statistics connection for the physical field can be derived from a suitable version of the "rubber band lemma" in the tail at infinity of Dirac dressing**

Comment on skyrmions

- Skyrmions (Skyrme 1962) are solitons of an SU(3)- NLσ model, carrying topological “baryonic “ charge and modeling baryons in QCD.
- Soliton worldlines can be considered as dual to particle worldlines and they carry a topological flux. At the boundary of open worldlines, where solitons are created/annihilated, a dual Dirac ansatz provides current distributions through which the topological flux spread out. For skyrmions these distributions (analogue of E of original Dirac ansatz) are represented by instantons of zero size.
- Solitons correlators are constructed coupling the dynamical fields of the NLσ model to such instanton distributions -> spin/statistics transmutation extending beyond the semi-classical approximation the results of Witten (1983).

Anyon non-local field operators

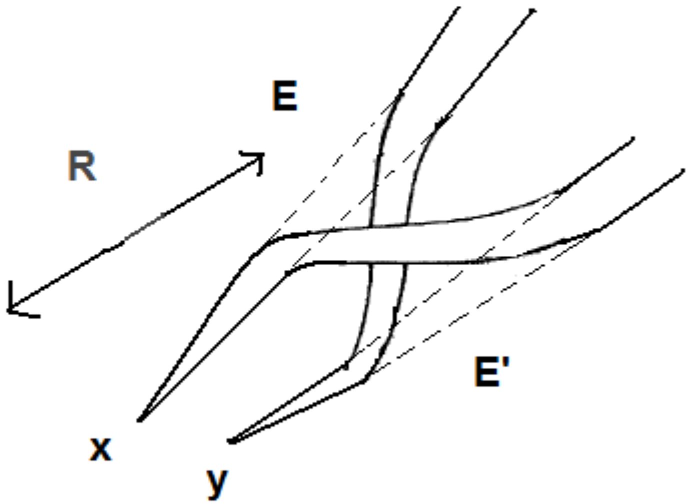
- The particle spectrum of the theory is massive (Chern -Simons term makes the photon massive) -> Buchholz-Fredenhagen (1982) theorem: in RQFT the support of the physical non-local field operators "creating" particles (hence $\hat{\phi}(\mathbf{E_x})$) can be chosen inside a space-like wedge.
- For non-overlapping supports such wedges can be ordered imposing origin and orientation to the space of space-like directions at infinity $\approx S^1$.



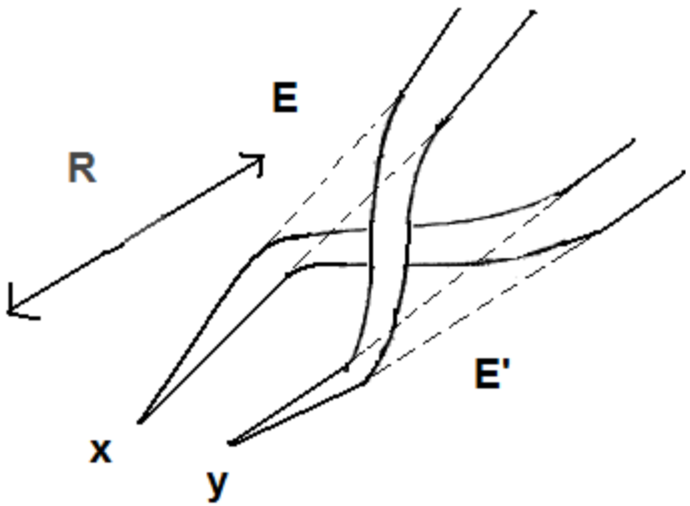
Exchanges with orientation

- This order allows to distinguish the orientation of the exchange : $+\sigma \leftrightarrow$ overcrossing (- $\sigma \leftrightarrow$ undercrossing) if the order in the product of the field operators agree (disagree) with the order of the directions at ∞ of their support.
- $\hat{\phi}(\mathbf{E}_x)\hat{\phi}(\mathbf{E'}_y)=$
  $\lim_{R\to\infty} U(+\sigma)^R \hat{\phi}(\mathbf{E'}_y)\hat{\phi}(\mathbf{E}_x)=$
  $\exp[i\,\pi/k]\, \hat{\phi}(\mathbf{E'}_y)\hat{\phi}(\mathbf{E}_x)$

  $\hat{\phi}(\mathbf{E'}_y)\hat{\phi}(\mathbf{E}_x)=$
  $\lim_{R\to\infty} U(-\sigma)^R \hat{\phi}(\mathbf{E}_x)\hat{\phi}(\mathbf{E'}_y)=$
  $\exp[-i\,\pi/k]\, \hat{\phi}(\mathbf{E}_x)\hat{\phi}(\mathbf{E'}_y)$



$U(+\sigma)^R \hat{\phi}(\mathbf{E'}_y)\hat{\phi}(\mathbf{E}_x)$



$U(-\sigma)^R \hat{\phi}(\mathbf{E}_x)\hat{\phi}(\mathbf{E'}_y)$