Spacetime and Noncommutative Geometry

Trieste

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This public lecture is an interlude to what has been happening here during this week.

We have been hearing challenges to the fundamental principles governing physics, chemistry and indeed life.They question the nature of spacetime, causality as we understand it, principles of relativity and other cherished principles.

This is dangerous talk, and may be perceived even as the rantings of reckless revolutionaries.

I want to convince you that it is not so.

We are sober physicists, not drunk at all.

Rather these radical ideas are naturally suggested when we examine nature in the small.

ON SPACETIME IN THE SMALL

There are good reasons to imagine that spacetime at the smallest scales will display some sort of discreteness.

They come from *quantum physics* and *the nature of black holes*.

Let me outline them.

The length scales in gravity are far smaller than all nuclear or atomic scales, being 10^{-32} centimetres. This is 10^{19} times smaller than the size of an atom.

At such small length scales, there is no reason to suppose that the nature of spacetime would be anything like what we are familiar with.

In fact, there are good reasons to suppose that it would be radically different.

In particular we expect fundamental limitations on probing such small length scales. This comes about from considerations involving gravity and quantum theory.

The following arguments were described by Doplicher, Fredenhagen and Roberts.

PROBING PLANCK LENGTH-SCALES

In order to probe physics at the Planck scale $L \simeq 10^{-32} cms$, the Compton wavelength λ_C of the probe of mass M must therefore fulfill

$$\lambda_C = \frac{\hbar}{Mc} \leq L \;\; or \;\; M \geq \frac{\hbar}{Lc} \simeq \text{Planck mass} \simeq 10^{19}\text{GeV} \simeq 10^{-5}\text{grams.}$$

Such huge mass in the small volume $L^3 \simeq (10^{-32} cm)^3$ will strongly affect gravity and can cause *black holes* and their *horizons* to form: gravity can become so large that no signal can escape the volume.

LESSON FROM QUANTUM MECHANICS

Quantum theory too puts fundamental limitations on measurements of position x and momentum p:

$$\Delta x \Delta p \geq \hbar/2$$

Δx, Δp = uncertaintities in position and momentum measurements.

There is a precise mathematical manner to achieve this [Schrödinger, Heisenberg, Dirac, ...]:

Make x and p “noncommuting”:

$$xp - px = i\hbar.$$

This suggests we can incorporate limitations on spatial resolution by making position noncommuting as in

$$xy - yx = i\theta$$

where θ is a new Planck-like fundamental constant with dimension $(\text{length})^2$.

When spacetime is noncommuting, we say that it obeys “noncommutative” geometry.

Now

$$\Delta x \Delta y \geq \theta/2$$

where Δx, Δy = intrinsic uncertaintities in x and y coordinate measurements.

There are similar equations in any pair of coordinates.

THE MOYAL PLANE

The Groenewold - Moyal (G-M) plane is the algebra of functions $\mathcal{A}_\theta$ on $\mathbb{R}^{d+1}$ with a twisted product:

$$f * g(x) = f \, e^{i/2 \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} \, g.$$

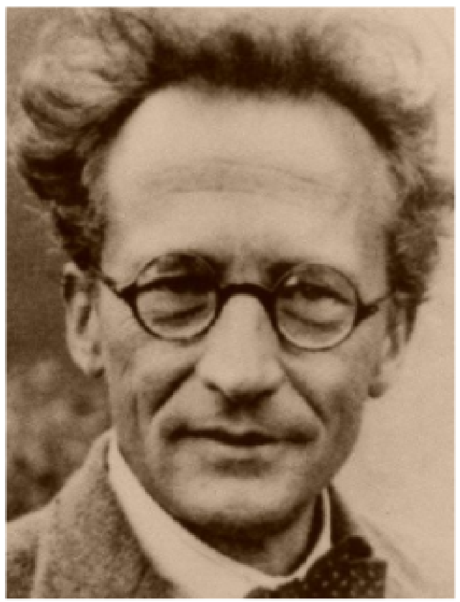
It implies that spacetime is noncommutative:

$$\widehat{x}_\mu \star \widehat{x}_\nu - \widehat{x}_\nu \star \widehat{x}_\mu = \left[\widehat{x}_\mu, \widehat{x}_\nu\right]_\star = i\theta_{\mu\nu}, \quad \mu, \nu = 0, 1, \cdots, d.$$

$$\widehat{x}_\mu = \text{coordinate functions}, \quad \widehat{x}_\mu(x) = x_\mu.$$

A BIT OF HISTORY

The idea that spacetime geometry may be noncommutative is old. It goes back to Schrödinger and Heisenberg.



Erwin Schrodinger

Werner Heisenberg

Heisenberg raises this possibility in a letter to Rudolph Peierls in the 30's.

Heisenberg also complains that he does not know enough mathematics to explore the physical consequences of this possibilty.

Peierls mentions Heisenberg's ideas to Wolfgang Pauli.

He in turn explains it to Hartland Snyder.

And it is Snyder who publishes the first paper on the subject in Physical Review in 1941/42.

I should also mention the role of Joe Weinberg in these developments. Joe was a student of Oppenheimer and was a close associate of Pauli and a classmate of Schwinger. He was the person accused of passing nuclear secrets to the Soviets and who lost his job in 1952 at Minnesota for that reason. His wife supported the family for several years. Eventually he got a faculty position at Case in 1958 and from there, he came to Syracuse.

Joe was remarkable. He seemed to know everything, from Sanskrit to non-commutative geometry, and published very little. He had done extensive research on this new vision of spacetime. He had showed me his manuscripts. They are now in Syracuse University archives.

QUANTUM FIELDS ON THE MOYAL PLANE

Pioneered by

- Doplicher, Fredenhagen, Roberts
- Julius Wess and his group

Wess was a student of Hans Thirring, father of Walter Thirring in Vienna.

Julius passed away in Hamburg in August, 2007 at the age of 72. All of us in this field miss him for his kindness and wisdom.

ON STANDARD QUANTUM FIELD THEORIES

– They treat particles as point particles, with no extension.

– They obey “causality”.

These features lead to profound consequences, such as

- The Pauli principle

- The CPT theorem

A Digression : What is causality

There are different, apparently unrelated, concepts.

KRAMERS-KRONIG RELATION

The system should not respond before the time at which it is disturbed. If $R(t)$ is response and disturbance of system is zero for time < 0,

$$R(t) = 0 \quad t < 0. \tag{1}$$

Its Fourier transform

$$\tilde{R}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega\, t} R(t) = \int_{0}^{\infty} e^{i\omega\, t} R(t) \tag{2}$$

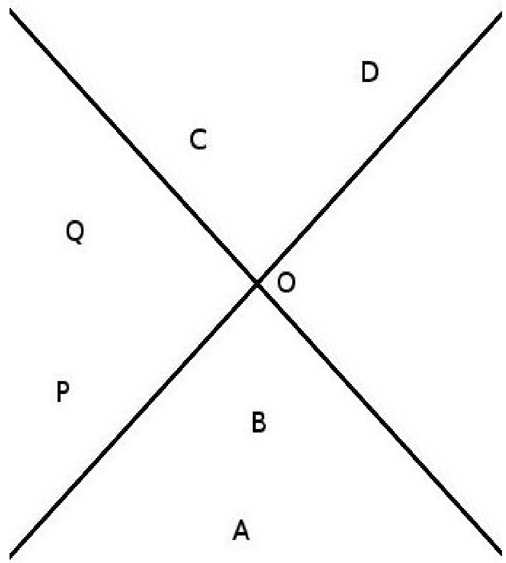
is holomorphic in

$$\Im\omega > 0. \tag{3}$$

SORKIN

Causality for events is a partial order $>$:

"$A > B$ if A is to future of B."



O is the observer. C, D are in the future, and A, B are in the past light cone of O. They are causally related to O. Q, P are spacelike relative to O and are not causally related to O.

Observables $\rho(x)$, $\eta(y)$ commute if x and y are spacelike separated:

$$[\rho(x), \eta(y)] = 0 \tag{4}$$

if

$$(x^0 - y^0)^2 - (\vec{x} - \vec{y})^2 < 0 \quad \text{or} \quad x \sim y. \tag{5}$$

Enforced by

$$[\varphi(x), \chi(y)]_- = 0 \quad x \sim y \tag{6}$$

for scalar fields,

$$\left[\psi_\alpha^{(1)}(x), \psi_\beta^{(2)}(y)\right]_+ = 0 \quad x \sim y \tag{7}$$

for spinor fields.

But these also express Statistics.

So Causality and Statistics are connected.

Moyal Plane describes noncommutative spacetime where commutation relations and hence causality and statistics are deformed.

ON NONCOMMUTATIVE QUANTUM FIELD THEORIES

– Particles have “extensions”

– Causality is violated. Signals can travel faster than light.

That is because light cone itself is not sharp: we cannot localize spacetime points.

So “inside” and “outside” light cone loses precise meaning.

So Pauli principle and CPT theorem are violated.

Consequences

Some of the consequences of noncommutativity are as follows:

1) Forbidden transitions can occur

2) CPT is violated

3) CMB spectrum is affected

4) There are correlations of observables localized at apce-like distances

5) Scattering cross-sections are not Loentz invariant

We briefly review each item

Forbidden Transitions

In the Borexino and SuperKamiokande experiments, the forbidden transitions from O^{16} (C^{12}) to $\tilde{O}^{16}$ ($\tilde{C}^{12}$) where the tilde nuclei have an extra nucleon in the filled $1S_{1/2}$ level are found to have lifetimes greater than 10^{27} years. There are also experiments on forbidden transitions to filled K-shells of crystals done in Maryland which give branching ratios less than 10^{-25} for such transitions.

From the data based on these experiments

$$\text{(Noncommutativity parameter)}^{\frac{1}{2}} \leq 10^{-24} \text{ cm},$$

$$\text{Energy scale} \geq 10^{11} \text{ GeV}.$$

CPT violation

Life times and magnetic moments of particles and anti-particles can differ.

K^0 and $\bar{K}^0$ can differ in mass.

Electron magnetic moment μ_e is the best measured number ever in science.

$$\mu_e \text{ (in units of } \frac{e\hbar}{2m_e c}) = 2.002319304.$$

It can differ from that of positron.

Same goes for muon where there are numbers for $\mu^{\pm}$ - magnetic moments

CMB data provide a third source to check noncommutativity.

Estimates of θ from such experiments are in progress.

Temperature fluctuation in CMB

In 1992, the COBE satellite detected temperature fluctuations (anisotropies) of the order of 10^{-5} in the CMB radiation.

It led to the conclusion that the early universe was not smooth: there were small perturbations in the radiation-matter fluid.

CMB radiation had its origin when the universe was much smaller in size than today.

So it can carry the signature of physics of the small scale - the noncommutative spacetime.

Anisotropies in the CMB can carry traces of spacetime noncommutativity in the early universe.

Expand temperature fluctuations in spherical harmonics:

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n}).$$

Then in the noncommutative framework, the angular correlation for two-point temperature fluctuations

$$C_l = \frac{1}{2l+1} \sum_m \langle a_{lm} a^*_{lm} \rangle$$

becomes "noncommutative": shows the effects of spacetime noncommutativity.

Estimates of the noncommutativity parameter from the CMB data give:

Length scale for noncommutativity $\lesssim 10^{-17}$ cms.

This corresponds to and energy scale $\gtrsim 10^{4}$ GeV.

We can see effects of noncommutativity only by probing nature at such high energies!

The Large Hadron Collider (LHC) operates around 10^{3} GeV. This energy scale is lower than the energy scale of noncommutativity.

SPACE-LIKE CORRELATIONS AND HOMOGENEITY PROBLEM

Let ω be a non-translationally invariant state density matrix (Eg: Friedman-Robertson-Walker state).

Consider a disturbance in Hamiltonian,

$$\delta H_I = \int N(y) A_0(y) d^3y, \quad y \in D_1,$$

in a spacetime region D_1:

$$\begin{aligned} A_0(y) &= 0 \text{ if } y \notin D_1 \\ N(y) &= \text{number density} \end{aligned}$$

Then for $x \in D_2$, fluctuation $\delta N(x)$ is,

$$\mathrm{Tr}\ \omega\delta N(x) \equiv \omega(\delta N(x)) = i \int d^4y \theta(x^0 - y^0)\omega\Big([N(x), N(y)]\Big)A_0(y).$$

This is zero in causal theories if $D_2 \sim D_1$, since $[N(x), N(y)] = 0$.

But on Groenewold-Moyal plane,

$$\omega(\delta N(x)) = i \int d^4y \theta(x^0 - y^0)\omega\Big([N(x), N(y)]\Big)A_0(y) \neq 0.$$

May have applications to homogeneity problem on Groenewold - Moyal plane.

The Scattering Matrix

Lorentz Noninvariance:

The S-matrix is not Lorentz invariant. The reason is loss of causality:

Let H_I be the interaction Hamiltonian density in the interaction representation. The interaction representation S-matrix is

$$S = T \exp\left(-i \int d^4 x H_I(x)\right).$$

Noncommutative theories are nonlocal and violate the causality condition: this is the essential reason for Lorentz noninvariace.

The effect on scattering amplitudes is striking. They depend on total incident momentum $\overrightarrow{P}_{\text{inc}}$ through

$$\theta_{0i}\left(\overrightarrow{P}_{\text{inc}}\right)_i$$

So effects of $\theta_{\mu\nu}$ disappear in the center-of-mass system, or more generally if

$$\theta_{0i}\left(\overrightarrow{P}_{\rm inc}\right)_i = 0$$

But otherwise there is dependence on θ_{0i}.

The violation is of order $\theta_{0i}\left(\overrightarrow{P}_{\rm inc}\right)_i$ in cross sections.

REMARK: Even with noncommutativity $Z^0 \longrightarrow 2\gamma$ is forbidden in the approach of Aschieri et. al.

More generally,

A massive particle of spin j does not decay into two massless particles of same helicity if j is odd.

CPT :

The effect of P and CPT is to reverse the sign of θ_{0i}:

$$\text{P or CPT} : \theta_{0i} \rightarrow -\theta_{0i}.$$

The θ_{0i} contributes to P, and more strikingly, to CPT violation.

Particle-antiparticle life times can differ to order θ_{0i}:

$$\tau_{\text{particle}} - \tau_{\text{antiparticle}} \cong \theta_{0i} \left(\overrightarrow{P}_{\text{inc}} \right)_i.$$

$(g-2)$ of μ^-, μ^+ can differ.

We are estimating bounds on θ from these effects.

FORMAL DEVELOPMENTS

Standard quantum field theories have many divergence problems because they treat particles as point-like.

In dramatic developments

Grosse and Wulkenhaar, Grosse and Wohlgenannt of Vienna and Münster, and the Paris group:

Gurau, Magnen, Rivasseau, Vignes-Tournaret, ...

have shown that

Many problems of standard quantum field theories disappear in noncommutative field theories.

This work can have enduring impact on fundamental physics.

FINAL REMARKS

With noncommutativity, there are violations of Pauli principle, Lorentz and CPT invariance, and effects on the CMB radiation.

Perhaps one can isolate characteristic signals of such violations and confront them with experiments.

One can put limits on noncommutativity even now from the observed CMB data and data on Pauli-forbidden transitions.

Important avenues of research have also been opened up in foundations of quantum field theory.

There are many interesting possibilities!