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# A philosopher's look at Pauli's exclusion principle

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# Introduction

- **Three main reasons why philosophers are fascinated by the Pauli principle**
    - **I. Metaphysics**
    - **II. Epistemology**
    - **III. The growth of scientific knowledge**
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# I. Metaphysics

- Pauli's exclusion principle and the problem of identity in physics

## A paradox

*“Roughly speaking, to say of two things that they are identical is nonsense, and to say of one thing that is identical with itself is to say nothing at all”.*

Wittgenstein, *Tractatus* 5.5303

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# Leibniz's principle of identity of indiscernibles

- $\forall a \forall b \forall F (F(a) \leftrightarrow F(b)) \rightarrow a = b$
- As a theorem of the predicate calculus, the principle of identity of indiscernibles (PII) states a logical truth, whose negation would be a contradiction



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- But PII is not just a logical truth within the predicate calculus nor was it originally meant to be such
  - It was rather originally formulated by Leibniz as a **metaphysical principle** about objects in natural world.
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## **Leibniz's New Essays (1704):**

*“I remember a great princess [the electress Sophia], of lofty intelligence, saying one day while walking in her garden that she did not believe there were two leaves perfectly alike. A clever gentleman who was walking with her believed that it would be easy to find some, but search as he might he became convinced by his own eyes that a difference could always be found”*

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## **Leibniz's Correspondence with Clarke (1716):**

*“I infer from that principle [principle of sufficient reason] among other consequences, that there are not in nature two real, absolute beings, indiscernibles from each other, because, if there were, God and nature would act without reason in ordering the one otherwise than the other; and that therefore God does not produce two pieces of matter perfectly equal and alike”*

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## Two main kinds of properties and two versions of PII

- i) two particulars must differ at least in some monadic properties: *Strong* PII
- ii) two particulars must differ at least in some relational properties: *Weak* PII.

**Both versions have been scrutinized and criticised in the light of Pauli's principle!**

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## Strong PII is not (contingently) true

- Leibniz used the principle to attack the hypothesis of indiscernible atoms. But already in **classical 19c. atomic physics**, the strong PII turned out to be false for atoms.
  - But what about **quantum mechanics**?
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- **H. Weyl** (1949) suggested that Pauli's exclusion principle in fact vindicated the validity of Leibniz's principle of identity of indiscernibles for fermions (there cannot be two fermions in the same dynamic state). The **'Pauli-Leibniz' principle!**
  - This claim has been challenged by a famous argument due to **Henry Margenau** (1950).
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# Margenau's argument against Weyl

- Pauli's principle, far from vindicating, would refute the validity of Leibniz PII.
  - The same reduced state, viz. an improper mixture, can be assigned as a separate state to each fermion of a composite system in antisymmetric state.
  - Hence two fermions **do** have the same monadic state-dependent properties (they are indiscernibles).
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## A follow-up on Margenau's argument

- Steven French (1989, 1995), French and Redhead (1988), Redhead and Teller (1992), French (2007) *Identity and individuality in physics* (OUP), Saunders and Muller (2008).
  - Massimi (2001) 'Exclusion principle and identity of indiscernibles. A response to Margenau's argument', *British Journal for Philosophy of Science* **52**, 303-330.
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## My point against Margenau

- A misleading assumption behind the argument. No separate states should be invoked since the states of the two particles are entangled and improper mixtures cannot be taken as **ontologically separate states**, hence cannot be taken as encoding monadic properties.
  - The strong version of PII is simply **inapplicable** to fermions.
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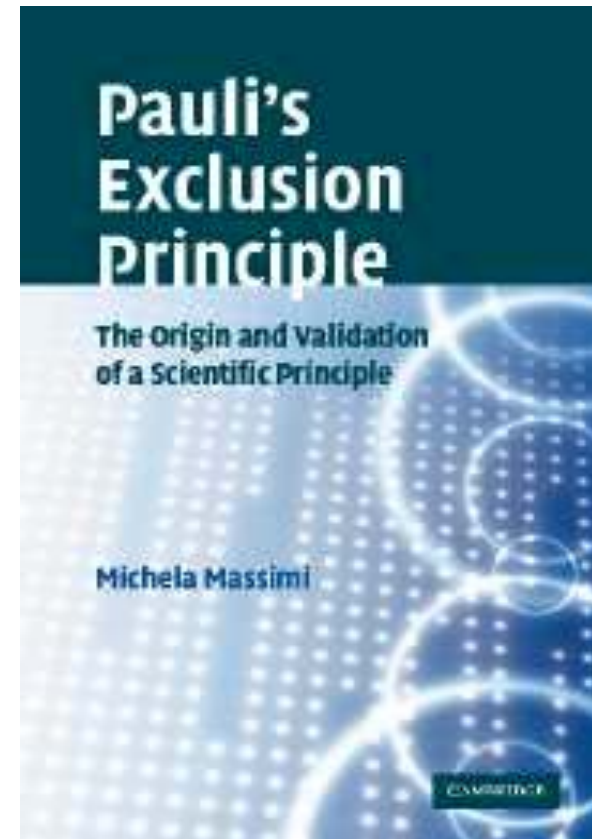
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## II. Epistemology

- What is a scientific principle? What is its origin? What makes a contingent empirical rule a 'principle'?
  - **Henri Poincaré (1905):** scientific principles are **useful conventions** that cannot be refuted by experiments.
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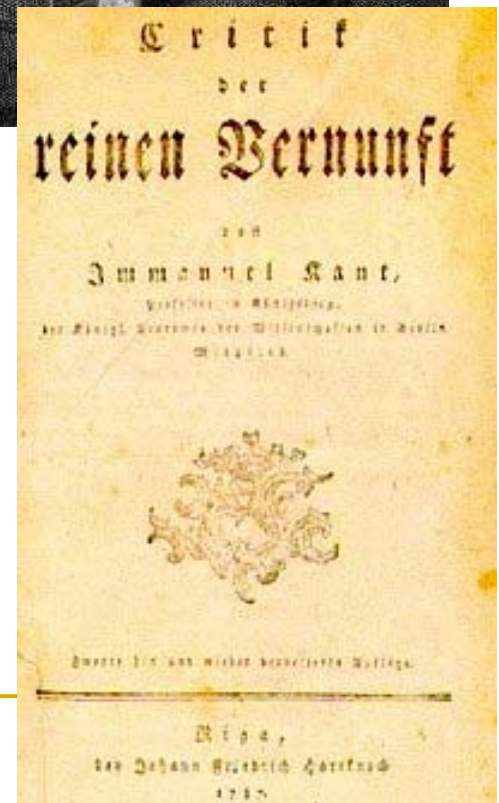
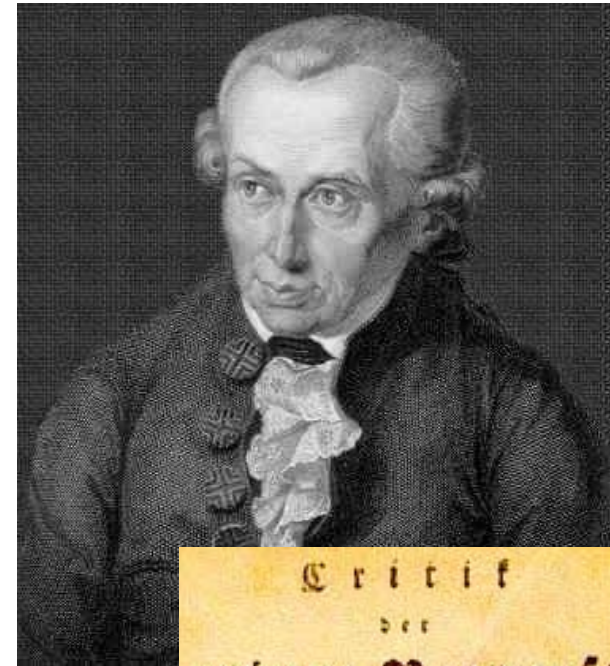
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- **Karl Popper** (1934) on Pauli's principle as an auxiliary hypothesis introduced to increase the degree of falsifiability of Bohr's atomic theory.
  - **Imre Lakatos** (1978) on Pauli's principle as 'invented' to shelter Bohr's atomic theory from possible negative spectroscopic evidence.
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- **My book** *Pauli's exclusion principle. The origin and validation of a scientific principle* (CUP, 2005).
  - Two main aims:
    1. To reconstruct the history of the principle from 1924 to recent experimental tests on possible violations
    2. To address the philosophical question above (what turns a contingent empirical rule into a fundamental scientific principle?)
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- EP is **neither** a useful convention (pace Poincaré)
- **Nor** is it an auxiliary assumption introduced to either increase the degree of falsifiability of Bohr's theory (Popper) or to shelter it from refutation (Lakatos).
- **I favor a Kantian approach** (after the German philosopher Immanuel Kant)



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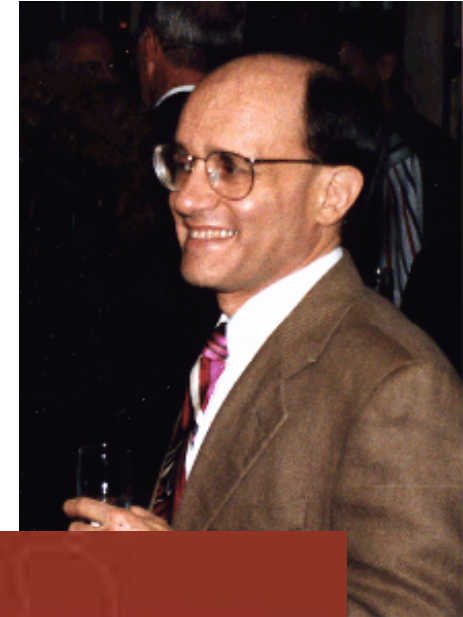
# Contemporary Kantian epistemology

Gerd Buchdahl

*Metaphysics and the Philosophy of Science* (1969)

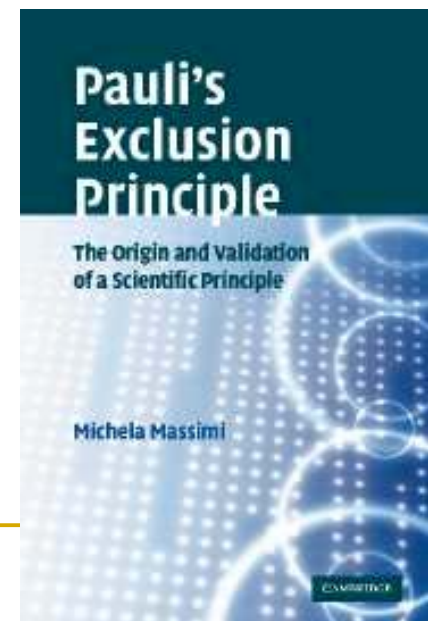
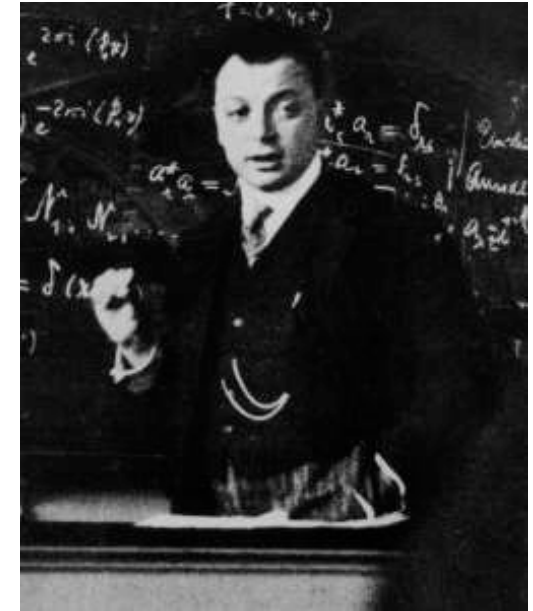
Michael Friedman

*The dynamics of reason* (2001):  
Kant's epistemology applied to  
twentieth-century physics



# My research

- Latching onto Friedman's dynamic Kantianism: an analysis of Pauli's exclusion principle
- How is our scientific knowledge as displayed by QM (and Pauli's principle in it) possible?



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## This Kantian view exemplified in the history of the Pauli's principle (ch. 2, 4)

- Pauli's *Ausschliessungsregel* embedded into a growing theoretical framework:
  - from the Fermi–Dirac statistics in 1926, to the non-relativistic quantum mechanics of Pauli's spin matrices in 1927
  - from Wigner and von Neumann's group theoretical derivation of the spin matrices in 1927, to Jordan's reinterpretation of Pauli's principle in terms of anticommutation relations for particle creation and annihilation operators in 1928.
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- The most important step in the building-up of this theoretical framework was the transition from non-relativistic to relativistic quantum mechanics, with Dirac's equation for the electron in 1928
  - The negative energy solutions of the Dirac equation, Dirac's attempt to accommodate them via the hole theory in 1930, and Pauli's sustained criticism of Dirac's hole theory eventually led Pauli in 1940 to the **spin–statistics theorem**.
  - With this theorem, the shift from a phenomenological rule to a well-entrenched principle, is finally completed.
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# Pauli's 1940 proof of the spin-statistics theorem

'Hence we come to the result: *For integral spin, the quantization according to the exclusion principle is not possible. (...)* On the other hand, it is formally possible to quantize the theory for half-integral spins according to Einstein–Bose statistics, *but (...)* the energy of the system would not be positive. Since for physical reasons it is necessary to postulate this, we must apply the exclusion principle in connection with Dirac's hole theory.'<sup>[1]</sup>

<sup>[1]</sup> Pauli (1940), p. 722. Emphasis in the original.

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- Ironically enough, Pauli ended up relying on his old enemy (the Dirac hole theory) to get the right spin–statistics connection for fermions. So, despite the remark that ‘*the connection between spin and statistics is one of the most important applications of the special relativity theory*’,<sup>[1]</sup> Pauli was **not** able to prove the theorem only on the basis of the relativistic requirement of microcausality, and he fell back on the dichotomy microcausality / positive energy.  
<sup>[1]</sup> *Ibid.*, p. 722.
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- A unified treatment of integral and half-integral spin particles under the only relativistic requirement of microcausality seemed to be unavailable to Pauli's generation. And the dichotomy microcausality / positive energy is still present in many textbooks (Greiner and Reinhardt 1996).
  - By contrast with this tradition, Weinberg 1964 proved the theorem using microcausality only.
  - For an analysis, see Massimi and Redhead (2003) 'Weinberg's proof of the spin–statistics theorem', *Studies in History and Philosophy of Modern Physics* **34**, 621–650.
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## III. The growth of scientific knowledge

- How is it possible to validate a scientific principle? (recall Poincaré)
  - How can a principle retain its validity in the face of possible recalcitrant evidence?
  - **Heuristic fruitfulness** of Pauli's principle in opening up new avenues of research (QCD in the 1960s).
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- The experimental discovery of the  $\Omega^-$  particle confirmed the validity of Gell-Mann's 'eightfold way' model, but it also provided negative evidence against quarks obeying the exclusion principle.
  - **Two alternative research programmes** emerged in the 1960s
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## Two alternative research programs:

- 1) **reject** the strict validity of the exclusion principle and assume that quarks obeyed *parastatistics* (Gentile 1940, Green 1953, Messiah and Greenberg 1964, Greenberg and Nelson 1977)
  - 2) **retain** the exclusion principle and combine it with recalcitrant evidence by introducing a further degree of freedom for quarks ('colour').
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# From a philosophical point of view

- It is thanks to the availability of **parastatistics research program** that experimental tests of Pauli's principle began to be explored.
- The philosopher of science **Thomas Kuhn**: Scientific revolutions are made possible when anomalies arise that cannot be accounted by the currently accepted scientific paradigm.



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## The heuristic fruitfulness of Pauli's principle: colored quarks

- It is thanks to the desire to retain the validity of Pauli's principle, that the auxiliary hypothesis of 'color' for quarks was introduced and it opened up new, undreamt-of avenue of research (with QCD).
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# The validation of Pauli's principle

- thanks to these two rival programmes, the exclusion principle has been **experimentally validated**.
  - The process of validation is **two-way street**:
    1. The **parastatistics programme** made it possible to predict possible violations of the principle in the context of atomic and nuclear physics, hence made it possible experimental tests.
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2. the introduction of the a new degree of freedom (**colour**) for quarks so as to reconcile the Pauli principle with negative evidence, turned out to be supported by a wide-ranging array of empirical evidence including i) evidence about the neutral pion decay rate as predicted by the Adler–Bell–Jackiw anomaly; (ii) evidence such as hadron-jet and glue-jet events, among many others.

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# Conclusion

Pauli's Nobel Prize lecture (1946), p. 215:

'The history of the exclusion principle is already an old one, but its conclusion has not yet been written'.

...and this is why, 85 years later, Pauli's principle still keeps physicists (and philosophers alike) puzzling about it!

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