SPIN-STATISTICS VIOLATION IN NEUTRINO PHYSICS.

A.D. Dolgov

INFN, Ferrara 40100, Italy
University of Ferrara, Ferrara 40100, Italy
ITEP, 117218, Moscow, Russia

International Workshop "Theoretical and experimental aspects of the spin-statistics connection and related symmetries"

Trieste, October 21-25, 2008

Based on the works:

1. A. Dolgov, A. Smirnov, Phys.Lett. B621, 1, 2005;

2. A. Dolgov, S. Hansen, A. Smirnov, JCAP 0506, 004, 2005;

3. A. Barabash, A. Dolgov, R. Dvornicky, F. Simkovic, A. Smirnov Nucl.Phys. B783, 90, 2007;

4. A. Dolgov, V. Novikov, M. Pospelov, in progress.

Content of the talk:

Phenomenological manifestations of (a large) admixture of bosonic component to partly fermionic neutrinos: cosmology, astrophysics, and particle physics. Difficult in absence of a consistent theory, only "resonable" guess.

An attempt to make a model of statistics violation by vacuum fermionic condensate.

Why neutrinos?

Because experimental bounds are weak and the effects might be LARGE.

More serioiusly:

The only known particle indicating new physics, beyond MSM:

1. Origin of neutrino mass.

2. Non-conservation of leptonic flavors (electronic, muonic, tauonic).

3. The only known particle for which Majorana mass is possible: breaking of the total leptonic number conservation.

Neutrinos may be messengers from hidden sector where our sacred principles are not respected and this could give rise to exotic/exciting possibilities:

breaking of CPT invariance

breaking of Lorentz invariance,

nonlocality, etc...

but hopefully causality survives.

MOST EXOTIC POSSIBILITY:

breaking of spin-statistics relation

Did Pauli invent a particle which breaks the Pauli exclusion principle?

If so, all above, CPT, Lorentz, etc may/must be broken too through (weak) communication with neutrinos.

Fermi, 1934: maybe electrons are a little bit not identical

LATER PAULI PRINCIPLE VIOLATION FOR "NORMAL" MATTER, ELECTRONS, NUCLEONS: Ignatyev, Kuzmin, Okun, Mohapatra, Greenberg, Govorkov...

Very strong upper bounds. Weak interactions of neutrinos makes it natural, if spin-satistics is broken by neutrinos.

Possible accomponying effects of spin-statistics violation:

NONLOCALITY

FASTER-THAN-LIGHT SIGNALS

BROKEN CPT

UNITARITY ???

NON-POSITIVE ENERGY, UNSTABLE VACUUM ???

THEORETICAL PROBLEMS OR INSPIRATION ?

OBSERVATIONAL SIGNATURES OF BOSONIC NEUTRINOS

1. Large scale structure of the universe: cold and hot dark matter made of KNOWN particles, neutrinos.
2. Big bang nucleosynthesis.
3. Neutrino spectrum from supernovae.
4. Z-burst model for ultrahigh energy cosmic rays (maybe out of interest).
5. Two neutrino double beta decay.
6. Statistics violation through neutrino communication to e, p, n.

Postpone non-existing theory and consider phenomenology of neutrinos obeying Bose or mixed statistics.

What can we buy for this price?

Interesting possibility to study.

Cosmological dark matter made of known particles, i.e. of neutrinos!

Observable effects in neutrino physics and maybe in usual matter..

Even without good theory some formalism is necessary. Introduce femionic, f and bosonic, b, annihilation operatos, $a = cf + sb$, and define one-neutrino state as:

$$|\nu\rangle = \hat{a}^{+}|0\rangle = c|f\rangle + s|b\rangle$$

where $c = cos\delta$ and $s = sin\delta$.

Two neutrino state:

$$|k_1, k_2\rangle = \hat{a}_1^{+}\hat{a}_2^{+}|0\rangle$$

Postulate commutators:

$$\hat{f}\hat{b} = e^{i\phi}\hat{b}\hat{f}, \;\; \hat{f}^{+}\hat{b}^{+} = e^{i\phi}\hat{b}^{+}\hat{f}^{+},$$
$$\hat{f}\hat{b}^{+} = e^{-i\phi}\hat{b}^{+}\hat{f}, \;\; \hat{f}^{+}\hat{b} = e^{-i\phi}\hat{b}\hat{f}^{+}.$$

Two-neutrino decay amplitude:

$$A_{2\beta} = cos^2\chi\, A_f + sin^2\chi\, A_b,$$

where $cos^2\chi = c^4 + c^2s^2(1 - cos\phi)$ **and** $sin^2\chi = s^4 + c^2s^2(1 + cos\phi)$.

Warning: do not take it to seriously and do not go to multiparticle states. **It reminds effective Lagrangian when only first order is allowed. A convenient parametrization only.**

Double beta-decay - Barabash talk:

$$\boldsymbol{sin^2\chi < 0.7}$$

LSS AND DARK MATTER

Normal neutrinos cannot make cosmological cold dark matter for any spectrum of density perturbations and any interactions.
Tremain-Gunn limit: Fermi exclusion forbids too many neutrinos in a galaxy.
If m_ν is bounded by Gershtein-Zeldovich limit the galactic DM mass would be too small. (It is also true for light thermal bosons by another reason.)

The choice is either

NEW PARTICLES, OLD PHYSICS

or

OLD PARTICLES AND VERY NEW PHYSICS

Occam: "Plurality should not be posited without necessity".

BOSONIC NEUTRINOS CAN MAKE ALL OBSERVED COSMOLOGICAL DARK MATTER, COLD AND HOT.

They should form Bose condensate. A large lepton asymmetry,

$$|n_\nu - n_{\bar{\nu}}|/n_\gamma \sim 100$$

is necessary. It may be created in a version of Affleck-Dine model.

Problems with BBN?

Equilibrium distribution for purely bosonic neutrinos:

$$f_{\nu_b} = \frac{1}{\exp[(E - \mu_\nu)/T - 1]} + C\delta(k)$$

If chemical potential $\mu_\nu = m_\nu$ (maximum allowed value) and lepton asymmetry is large then ν_b should condense, i.e. $C \neq 0$, and become COLD.
To make all DM we need $C \sim 10 T_\nu^3$.

With $m_\nu = 0.1$ eV neutrinos would make CDM if

$$n_\nu \sim 10^4 \text{ cm}^{-3}$$

It is TWO ORDERS of magnitude larger than the conventional number.

In galaxies the neutrino number density could be about 6 orders of magnitude higher

$$n_\nu^{(gal)} \sim 10^{10} \text{cm}^{-3}.$$

Cosmological neutrino condensate may explain anomalies in tritium experiments for search of neutrino mass and oscillations of the life-time of nuclei!?

To this end neutrino number density around the Earth should be about $10^{16} - 10^{17}/\mathrm{cm}^3$ and $m_\nu < 10^{-7}$ eV.

Since double beta decay excludes 100% bosonic neutrinos the numbers would be somewhat different.

Kinetics with mixed statistics.

Kinetic equation (standard):

$$F = f_1(p_1)f_2(p_2)[1 \pm f_3(p_3)][1 \pm f_4(p_4)] \\ - f_3(p_3)f_4(p_4)[1 \pm f_1(p_1)][1 \pm f_2(p_2)]$$

How mixed statistics can be described?

Wild guess (justified aposteriori by the nice result):

$$(1 - f_\nu) \to c^2(1 - f_\nu) + s^2(1 + f_\nu)$$

Another possibility:

$$(1 - f_\nu) \to c^2(1 - c^2 f_\nu) + s^2(1 + s^2 f_\nu).$$

In both cases $(1 - f_\nu) \to (1 - \kappa f_\nu),$

$$\kappa = c^2 - s^2$$

is Fermi-Bode mixing parameter. The angle γ in $c = cos\gamma$ and $s = sin\gamma$ is not necessarily the same as in 2β amplitude introduced above.

EQUILIBRIUM DISTRIBUTION:

$$f_\nu^{(eq)} = \left[\exp\left(E/T\right) + \kappa\right]^{-1} .$$

κ runs from $+1$ (Fermi) to -1 (Bose); $\kappa = 0$ (Boltzmann).

Maximum chemical potential for which condensation occures:

$$\mu^{(max)} = m_\nu - T \ln\left(- \kappa\right)$$

Bose condensation can take place for negative κ only.

EFFECTS ON BBN.

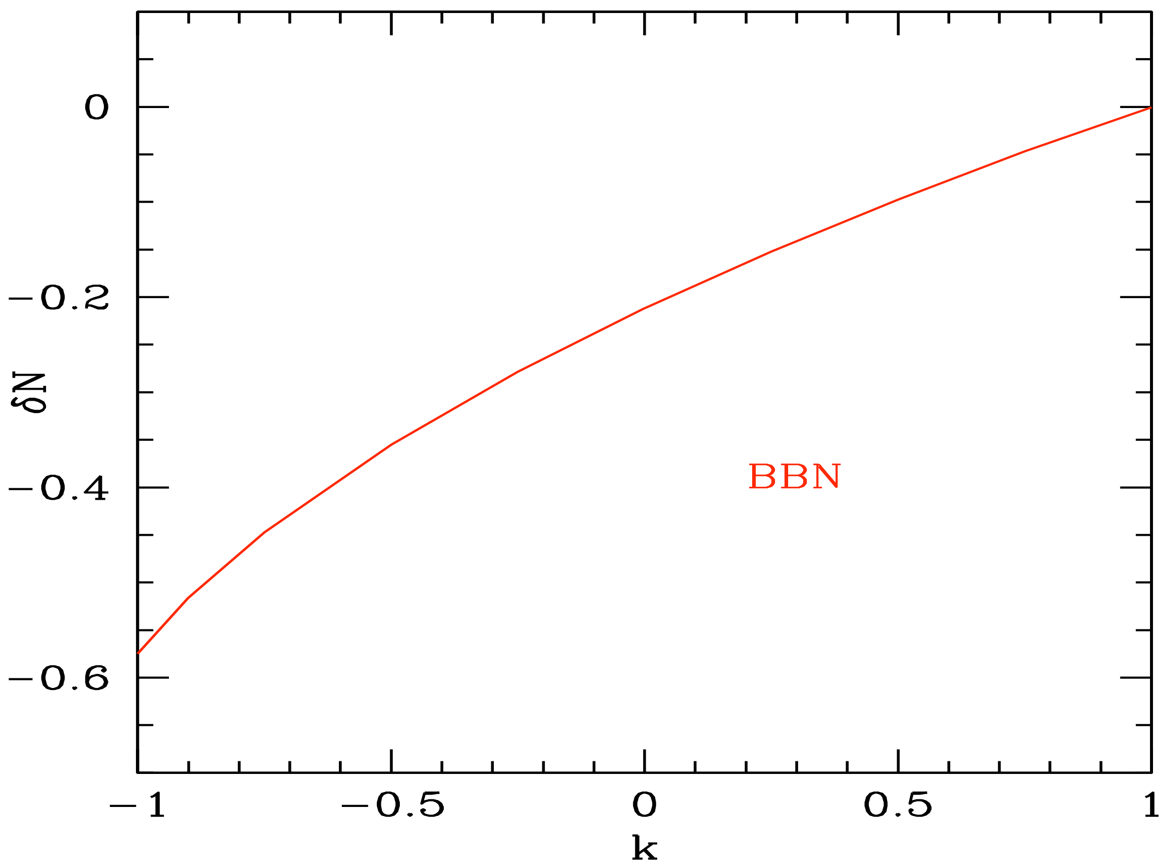
L. Cucurull, J.A. Grifols, R. Toldra. Aspropart.Phys. 4 (1996) 391; A. Dolgov, S. Hansen, A. Smirnov.

1. Larger energy density of ν:
$N_\nu^{(eff)}$ rises.

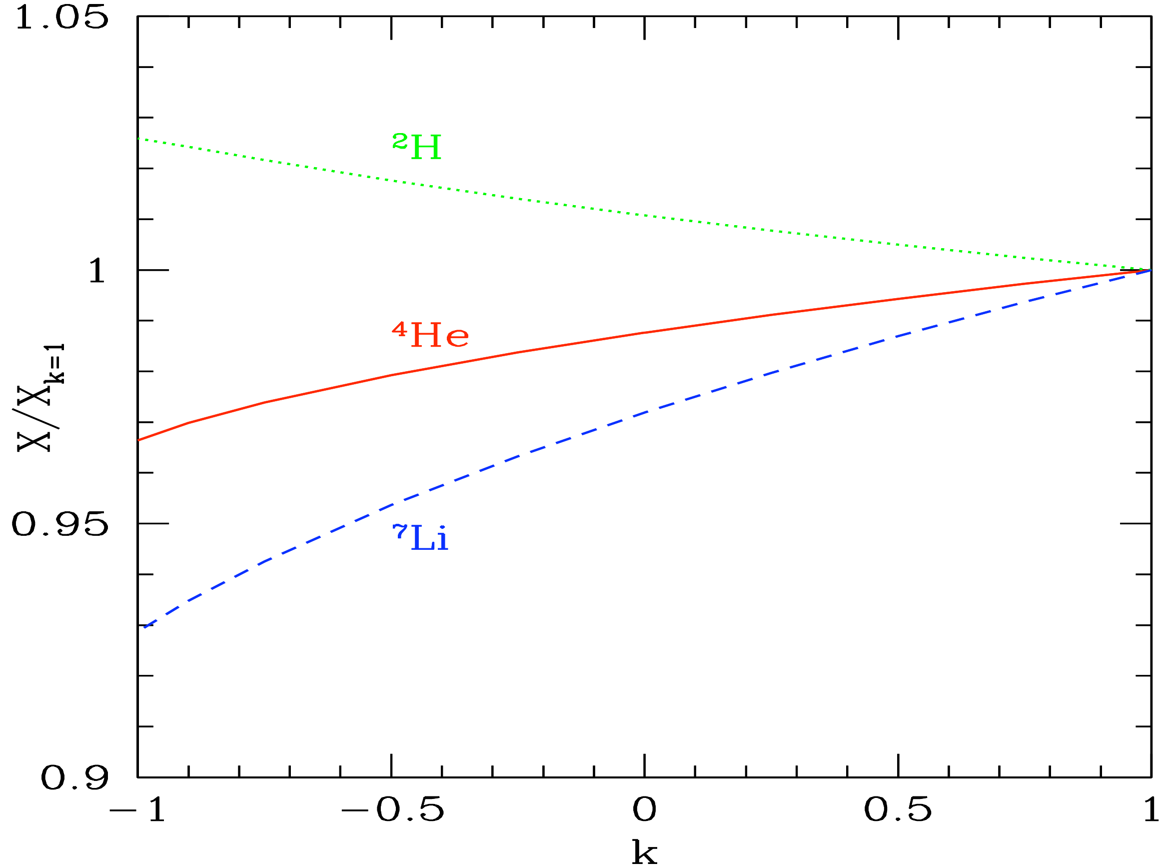
2. Larger rate of neutron-proton transformations, $N_\nu^{(eff)}$ drops.

Second effect dominates and

$$N_\nu^{(eff)} < 2.43.$$



WITH ZERO OR NEGLIGIBLE CHEMICAL POTENTIAL OF NEUTRINOS.



Better agreement with the data if $\kappa < 0$.

BBN limits on chemical potential and Bose condensation.

In the standard case:

$$|\xi_a| \equiv |\mu_a/T| < 0.07$$

for any ν_a.

If there is coupling to light majoron:

$$|\xi_e| < 0.1,\ |\xi_{\mu,\tau}| < 2-3.$$

In presence of nu-condensate:

$$f = \frac{1}{\exp(E-\mu)/T - \kappa} + C\delta^3(p)$$

$$1 - f \to 1 + \frac{\kappa}{e^{(E-\mu)/T} - \kappa} + C\delta^3(p)$$

For bosonic ν the effect of $\xi \approx ln|\kappa|$ may be canceled by condensate C, which changes the expansion rate and leads to higher n/p freezing temperature (preliminary):

$$(n/p)_{eq} = e^{-(\Delta m + \mu_{\nu_e})/T}$$

Bosonic neutrinos from supernovae.

Energy spectrum of ν shold be more narrow.

Violation of the Pauli principle allows for a smaller chemical potential of bosonic ν. This leads to a faster cooling and lower central temperature.

Neutrino induction of spin-statistics violation in ordinary matter.

Bosonic component of electron:

$$e \to W + \nu \to e$$

mass renormalization and may be unobservale (?).

In external electric field, naive estimate:

$$sin\gamma \sim (\alpha/2\pi)\,(m_e/m_W)^2 \sim 10^{-13}.$$

For nucleons the effect should be suppressed by 2nd order in weak interactions.

Some theoretical comments, trivial or incorrect.

Scattering matrix:

$$S = 1 + \sum_n \frac{(-i)^n}{n!} \int \Pi d^4 x_j \; T\{\mathcal{H}(x_1)...\mathcal{H}(x_n)\}$$

Lorentz invariant if $\mathcal{H}$ are bosonic. For bosonic ν amplitudes are not bosonic, even for pure statistics, e.g. for $e + p \leftrightarrow n + \nu$.

For mixed statistics amplitudes are not bosonic for any processes with neutrinos.

LORENTZ INVARIANCE MAY BE BROKEN.

Unitarity is probably maintained if $\mathcal{H}$ is hermitian.

Usually all fermions enter all observable quantities in even number. If not, observables do not commute and locality would be broken.

ALL THESE EFFECTS APPEAR IN HIGHER ORDERS ONLY.

Maybe Hamiltonian/Lagrangian approach is not applicable?
Or least action principle is A LITTLE violated?
If so, we do not have any formalism.

Fermion condensate

Introduce a source for fermions:

$$\boldsymbol{L \longrightarrow L + \bar{J}\psi + \bar{\psi}J,}$$

where $J, \bar{J}$ are "classical" currents for fermions, i.e. Grassman numbers.

In the case of $J \neq 0$ a nonzero current generates nonzero expectation value of the fermionic field:

$$< \psi >_J = \xi \neq 0,$$

where ξ is also a Grassman number. Nonzero ξ violates Lorentz and rotational symmetry.

Maybe it is possible to have nonzero value of ξ at zero current $J = 0$.

A mechanism for vacuum condensation of ξ, is unknown. In the standard QFT in four dimension it is impossible. But we can think about our space-time as a four-dimensional brane in multi-dimensional space with fermionic zero mode living on the brane. Another possibility is that the Creator forgot to switch-off the fermionic current J.

Consider a QFT model.

$$L = \bar{\psi}(\hat{p} - m)\psi + \phi(\hat{p}^2 - m^2)\phi/2 + \lambda\phi(\bar{\psi}\psi) + \bar{J}\psi + \bar{\psi}J$$

where $\phi(x)$ and $\psi(x)$ are neutral boson and fermion fields. Equations of motion looks like

$$(\hat{p} - m)\psi + \lambda\phi\psi + J = 0,$$
$$(\hat{p}^2 - m^2)\phi + \lambda(\bar{\psi}\psi) = 0.$$

For classical nonzero constant current

$$J(x) \equiv J = m\xi \neq 0$$

we get that

$$< \psi >_J \equiv \xi, \;\; < \phi >_J \equiv \frac{\lambda}{m^2}\bar{\xi}\xi$$

The propagation of the excitations in the vacuum with these two condensates

$$\psi = \xi + \psi_q; \quad \phi = \frac{\lambda}{m^2}\bar{\xi}\xi + \phi_q,$$

is described by the quadratic form

$$L^{(2)} = \bar{\psi}_q(\hat{p} - \bar{m})\psi_q + \frac{1}{2}\phi_q(\hat{p}^2 - m^2)\phi_q + \lambda\phi_q[\bar{\xi}\psi_q + \bar{\psi}_q\xi],$$

where $\bar{m} = m - \lambda^2\bar{\xi}\xi/m^2$.

The last term proportional to λ describes inelastic scattering on the fermionic condensate that transforms fermions into bosons and visa verse:

$$\boldsymbol{Bosons \Longleftrightarrow Fermions}$$

Quantum mechanical model

Expand field operators $\phi(x)$ and $\psi(x)$ as plane waves in a box

$$\phi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}}[a(p)e^{(ipx)} + h.c.]$$

$$\psi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}}[b(p)u(p)e^{(ipx)} + h.c.],$$

where $\omega^2 = \mathrm{p}^2 + m^2$, and $(a(p), a^+(p))$ and $(b(p), b^+(p))$ are annihilation and creation operators for the original scalar and spinor fields.

For the mode with given 3-momenta p we have a system with two degrees of freedom, i.e. simple Quantum Mechanics

$$H = \omega(p)[aa^+ + bb^+] + \lambda[a^+\zeta^+ b + b^+\zeta a],$$

with grasmanian parameter $\zeta = (\bar{u}\xi)/2\omega$ and operators that satisfy

$$[a, a^+]_- = [b, b^+]_+ = 1$$

All other (anti)commutators vanish.

This algebra is invariant under one-parameter group $(a, b) \longrightarrow (A, B)$:

$$a = [1 - \frac{1}{2}(\beta^* \beta)(\zeta^* \zeta)]A + \beta(\zeta^* B)$$

$$b = -\beta^* A \zeta + [1 + \frac{1}{2}(\beta^* \beta)(\zeta^* \zeta)]B,$$

where β is arbitrary complex.
Operators A, B satisfy the same c.r.

$$[A, A^+]_- = [B, B^+]_+ = 1.$$

To diagonalize the Hamiltonian we take

$$\beta = \beta^* = -\lambda/2\omega(p).$$

Now the Hamiltonian is the sum of bosonic and fermionic oscillators:

$$H = \omega_1 AA^+ + \omega_2 BB^+$$

with

$$\omega_{1,2} = \omega \pm \frac{\lambda^2 m}{16\omega^3}\bar{\xi}\xi,$$

Field theory.

In terms of the field variables

$$\phi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}}[a(p)e^{(ipx)} + h.c.],$$

$$\psi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}}[b(p)u(p)e^{(ipx)} + h.c.]$$

the transformation of operators $(a, b) \Rightarrow (A, B)$ is non-local.

$$\phi(x) \Rightarrow [1 - \frac{\lambda^2 m}{64}\bar{\xi}\xi\frac{1}{(-\nabla^2 + m^2)^2}]\phi(x) - \frac{\lambda}{4}\frac{1}{(-\nabla^2 + m^2)}[\xi\bar{\psi} + \psi\bar{\xi}],$$

$$\psi(x) \Rightarrow [1 + \frac{\lambda^2 m}{64}\bar{\xi}\xi\frac{1}{(-\nabla^2 + m^2)^2}]\psi(x) + \frac{\lambda m}{4}\frac{1}{(-\nabla^2 + m^2)}\phi(x)\xi.$$

By construction these non-local transformations do not violate causality.

Statistics

In terms of diagonal variables the spectrum of Hamiltonian is known and one can calculate the average number of particle at given state using the standard rules of Statistical Mechanics. For particles that are created by operator A^+ we get the canonical Bose distribution:

$$< N >_{Bose} = < AA^+ > = \frac{1}{e^{\omega_1/T} - 1}$$

with shifted frequency ω_1.

For particles that are created by operator B^+ we get the canonical Fermi distribution

$$< N >_F = < BB^+ > = \frac{1}{e^{((\omega_2 - \mu)/T)} + 1},$$

where μ is a chemical potential. These are the distributions for the diagonal states.

In terms of the initial particles that are created in collisions the same equations look like a mixed statistic . Indeed the distribution numbers for initial particles are

$$< n >_B = < aa^+ >, \; < n >_F = < bb^+ >,$$

we get

$$< n >_F = (1 + \beta^2 \zeta^+ \zeta) N_F - \beta^2 \zeta^+ \zeta N_B,$$

$$< n >_B = (1 - \beta^2 \zeta^+ \zeta) N_B + \beta^2 \zeta^+ \zeta N_F,$$

where $\beta = -\lambda / 2\omega$,

For the distribution of initial "neutrinos" we get

$$< n >_\nu = [1 + O(\lambda\bar{\xi}\xi)]\frac{1}{e^{((\omega-\mu)/T)} + 1} - \frac{\lambda^2 m}{32\omega^4}\bar{\xi}\xi\frac{1}{e^{(\omega/T)} - 1},$$

i.e. a sum of Fermi and Bose distributions. The admixture of Bose statistics is proportional to the condensation of Fermi field $\bar{\xi}\xi$.

Conclusion.

1. The suggestion of bosonic or mixed statistics for half integer spin particles opens a Pandora box of theoretical problems which may be impossible to solve without serious modification of the basic principles.

2. Such statistics leads to breaking of Lorentz invariance, CPT theorem, locality. Maybe causality and unitarity survive.

3. Bosonic neutrinos can make all cosmological DM.

4. BBN may be in a better agreement with the data.

5. Two-beta decay presents the best limit on the admixture of bosonic component to neutrinos but still very weak. There is an indication of better dscription of the data with a bosonic component of ν. Will it survive???

6. Induced by bosonic neutrinos violation of spin-statistics relation for the usual matter is difficult to evaluate but seems to be of primary importance.

7. We badly need a reasonable formalism for evaluation of effects of partly bosonic neutrinos. Maybe femionic condenstate opens such possibility.