In this test there are 2 exercises, the first is worth a maximum of 4 points, the second a maximum of 10 points.

1. Consider a photon beam with a Gaussian intensity profile, so that the delivered dose is given by the following equation

$$D(r) = D_B \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

where r is the distance from the beam axis and σ is a parameter that defines the beam width. The beam impinges on a disk of tumor cells, it is aimed at the center of the disk and its axis is perpendicular to the disk.

a) Describe the isodose curves on the disk, at $D = 0.5\ D_B$ and $D = 0.1\ D_B$. (3 points)

b) How do the isodose curves change if the beam is not perpendicular to the disk? (1 point)

2. Consider a tumor which has a total of N cells:

a) Recall the standard formula for the TCP. (1 point)

b) Recall the formula for the BED with total dose nD, delivered in n fractions. (1 point)

c) Modify the standard formula for the TCP to take into account a total dose nD which is split into n fractions. (2 points)

d) Consider the tables of alpha and beta coefficients reported in this scientific paper (link: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5956964/pdf/13014_2018_Article_1040.pdf), use the tables to find the mean values of alpha and beta for liver tumors (Tai 2008), the α/β ratio, and determine whether this is a late responding or early responding tissue. (2 points)

e) Use the standard formula for the TCP to find the dose that gives a 90% TCP, assuming that the tumor has $N = 10^9$ cells. (4 points)

Solutions

1. a) All isodose curves here are circles centered in the center of the tumor disk. The isodose curve $D = 0.5\ D_B$ is obtained from the solution of

$$D(r) = D_B \exp\left(-\frac{r^2}{2\sigma^2}\right) = 0.5\ D_B$$

i.e.,

$$\exp\left(-\frac{r^2}{2\sigma^2}\right) = 0.5$$

or, equivalently,

$$\frac{r^2}{2\sigma^2} = -\ln 0.5 \approx 0.6931$$

and the corresponding isodose curve is a circle with radius

$$r \approx \sigma\sqrt{2 \times 0.6931} \approx 1.18\ \sigma$$

Likewise, the isodose curve for $D = 0.1\ D_B$ is a circle with radius

$$r \approx 2.15\ \sigma$$

b) The isodose curves change into ellipses, where the minor axis is the same as the radius determined in the previous answer.

2. a) The standard formula for the TCP is

$$\mathrm{TCP} = \exp[-NS(D)] = \exp\left[-Ne^{-\alpha D - \beta D^2}\right]$$

b) The formula for the BED is

$$BED = nD\left(1 + \frac{D}{\alpha/\beta}\right)$$

c) The TCP for a fractionated therapy is

$$\mathrm{TCP} = \exp[-Ne^{-\alpha \mathrm{BED}}] = \exp\left[-Ne^{-\alpha n D\left(1+\frac{D}{\alpha/\beta}\right)}\right]$$

d) The tables in the paper return the following mean values for liver tumors

$$\alpha = 3.7 \times 10^{-2}\ \mathrm{Gy}^{-1}; \quad \beta = 2.8 \times 10^{-3}\mathrm{Gy}^{-2}$$

and $\alpha/\beta \approx 13.4\ \mathrm{Gy}$, so that this is an early-responding tissue.

e) The solution is found by solving this equation

$$\mathrm{TCP} = \exp[-NS(D)] = \exp\left[-Ne^{-\alpha D-\beta D^2}\right] = 0.9$$

i.e., taking logarithms

$$-Ne^{-\alpha D-\beta D^2} = \ln 0.9 \approx -0.1054$$

Using the value $N = 10^9$, we find

$$e^{-\alpha D-\beta D^2} \approx 1.054 \times 10^{-10}$$

and finally

$$\alpha D + \beta D^2 \approx 23.0$$

We find the dose by solving the quadratic equation

$$\beta D^2 + \alpha D - 23.0 = 0$$

and the (positive) solution is $D \approx 84.3$.