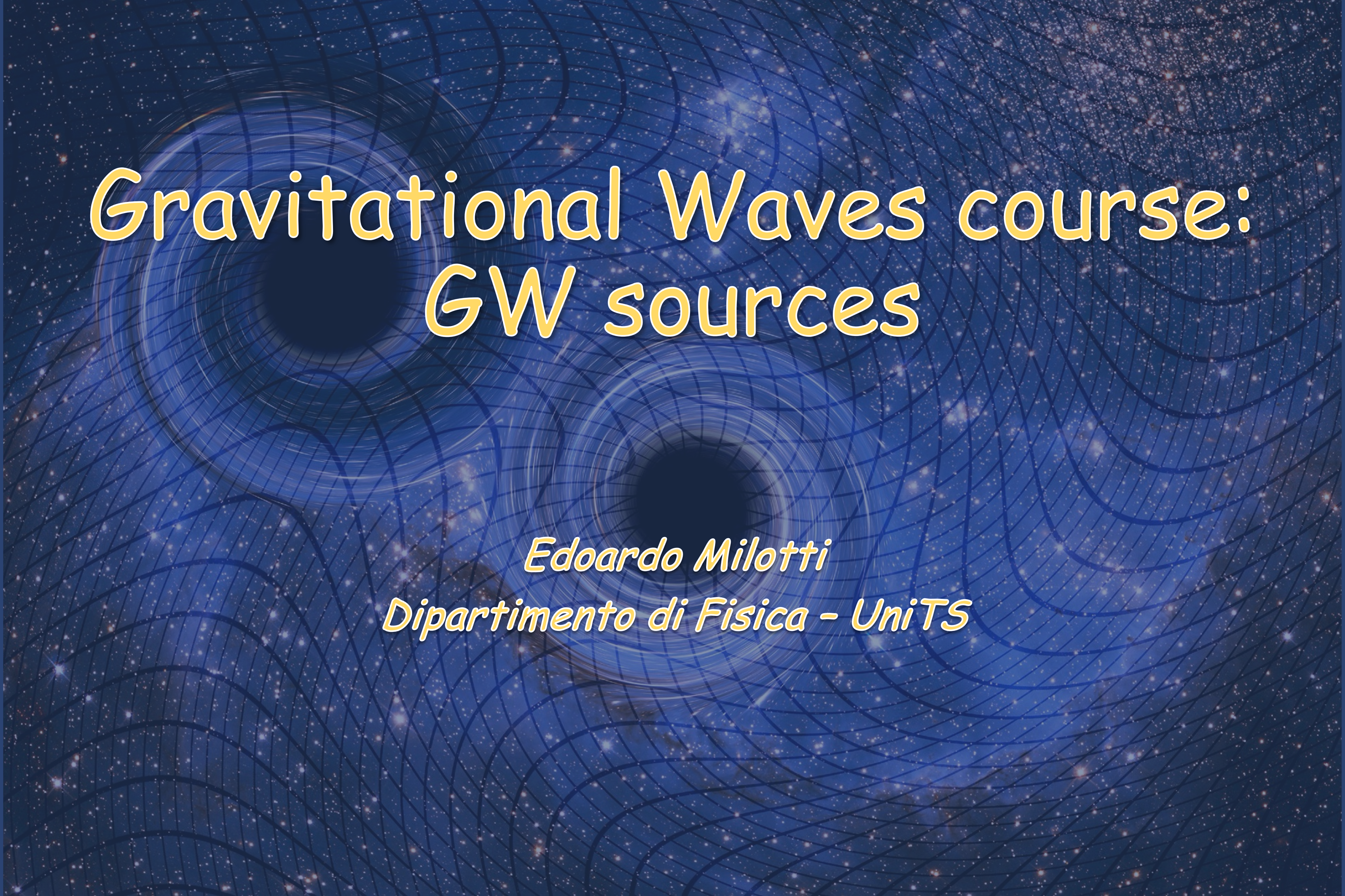


Gravitational Waves course: GW sources

Edoardo Milotti
Dipartimento di Fisica - UniTS

General remarks

- highest possible frequency for a system of given size

characteristic length

$$\Delta t \sim \frac{R}{v} > \frac{R}{c} \quad \Rightarrow \quad f \sim \frac{1}{\Delta t} < \frac{c}{R}$$

Examples:

$R = 10^6$ m $\Rightarrow$ $f < 300$ Hz

$R = 10^9$ m $\Rightarrow$ $f < 0.3$ Hz

$R = 10^{12}$ m $\Rightarrow$ $f < 300$ µHz

- frequency of dominant gravitational radiation, from dimensional analysis

$$f \sim \sqrt{\frac{GM}{R^3}} \sim \sqrt{G\rho} \quad \Rightarrow \quad \rho \sim 1.5 \times 10^{14} \left(\frac{f}{100\ \mathrm{Hz}}\right)^2$$

mass density

Examples:

$\rho = 2.8\ 10^{17}$ kg/m^3 $\Rightarrow$ $f \sim 4$ kHz

$\rho = 10^9$ kg/m^3 $\Rightarrow$ $f \sim 0.25$ Hz

$\rho = 10^3$ kg/m^3 $\Rightarrow$ $f \sim 250$ µHz

neutron star
nuclear matter

white dwarf
(1 solar mass)

density of water

General remarks (ctd.)

- Energy flux carried by gravitational waves

  (see also the handout on *The stress-energy tensor of gravitational waves and the energy flux*, eqs. (7)-(8))

$$\text{energy flux} = \frac{c^3}{16\pi G}\langle \dot{h}_+\dot{h}_+ + \dot{h}_\times \dot{h}_\times \rangle$$

Therefore, for a gravitational wave with frequency f and circular polarization, with amplitude h,

$$\text{energy flux} = \frac{c^3\pi f^2}{4G}h^2$$

Then for a burst of duration τ, with total energy E, at distance r, we find

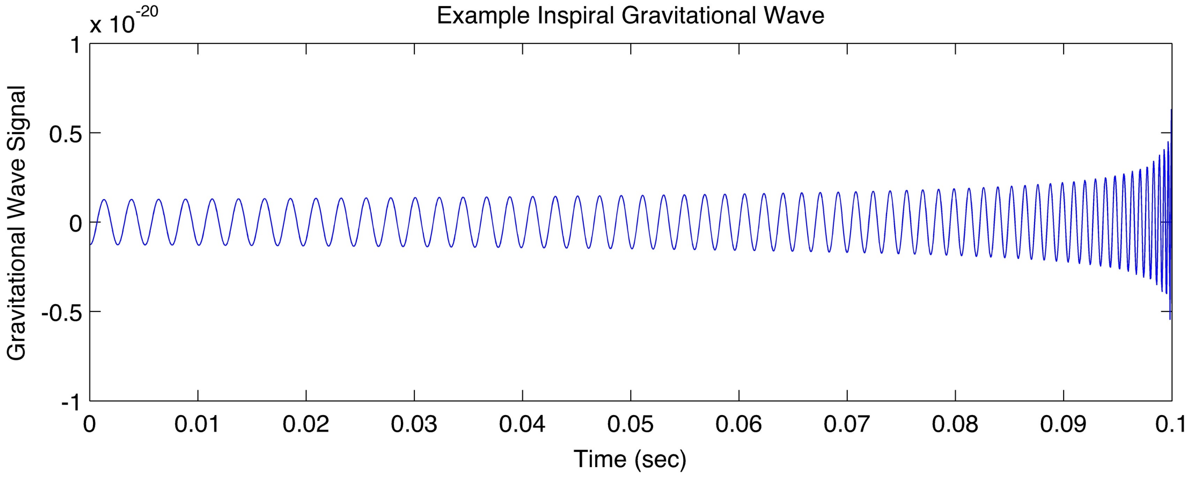
$$E = 4\pi r^2\tau\left(\frac{c^3\pi f^2}{4G}h^2\right) \quad \Rightarrow \quad h = \sqrt{\frac{G}{\pi^2 c^3}} \times \left(\frac{1}{fr}\sqrt{\frac{E}{\tau}}\right)$$

$$h = \left(\frac{G}{\pi^2 c^3}\right)^{1/2} E^{1/2} \tau^{-1/2} f^{-1} r^{-1}$$

$$h \approx 3.8 \times 10^{-19} \left[\frac{E}{M_\odot c^2}\right]^{1/2} \left[\frac{\tau}{1\ \mathrm{s}}\right]^{-1/2} \left[\frac{f}{1\ \mathrm{kHz}}\right]^{-1} \left[\frac{r}{1\ \mathrm{Gpc}}\right]^{-1}$$

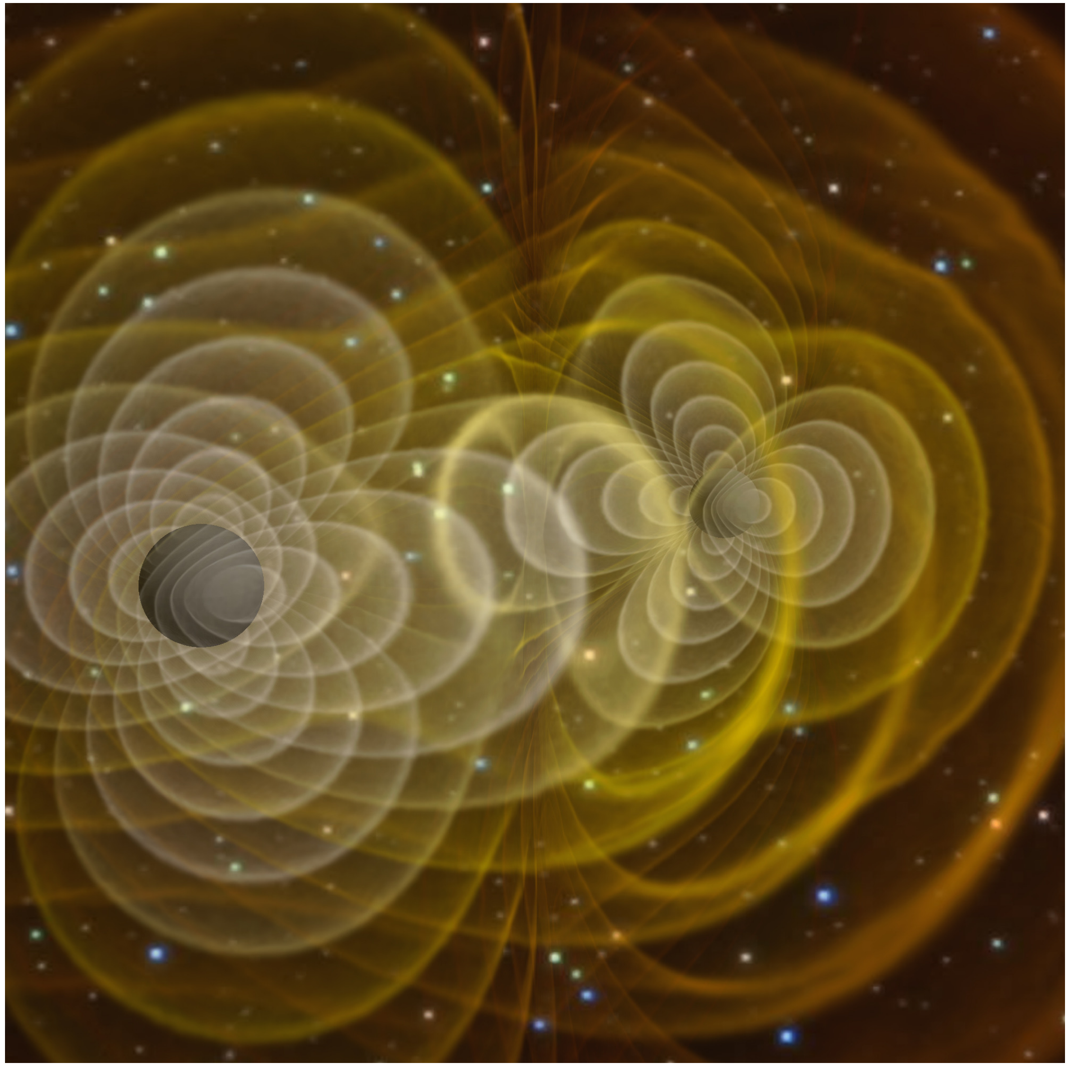
⬅ Distance to Virgo cluster is about 18 Mpc

Compact binary coalescences (inspiral phase)



Signal from a BBH coalescence

Signal from a BBH coalescence + actual noise



3D visualization of gravitational waves produced by a binary black hole. [Image: Henze, NASA]

Much material here is taken from https://www.ligo.org/science/GW-Sources.php
and https://www.ligo.caltech.edu/page/gw-sources

There are three subclasses of "compact binary" systems in this category:

- Binary Neutron Star (BNS)
- Binary Black Hole (BBH)
- Neutron Star-Black Hole Binary (NSBH)

Each binary pair creates a unique pattern of gravitational waves, but the mechanism of wave-generation is the same across all three.

The masses of the objects involved dictate how long they emit detectable gravitational waves. Heavy objects, like black holes, move through their final inspiral phase much more rapidly than 'lighter' objects, like neutron stars.

This means that black-hole merger signals are much shorter than neutron star merger signals, and the differences are quite striking. For example, the first pair of merging black holes that were detected produced a signal just *two-tenths of a second* long. In contrast, the first neutron star merger detected in August 2017 generated a signal over 100 seconds long in our instruments.

The GW interferometers convert the space-time distortion signals into an audible sound called a "chirp" so we can all, in a sense, 'hear' the final moments of the lives of two black holes and two neutron stars.

The objects in the first detected CBC signal had been orbiting each other for billions of years; the video below captures the last fraction of a second or few seconds of that lifetime together. The signal is played several times, repeating the chirp first in its natural frequency--the low 'thump'--and then increased to make it easier to hear.

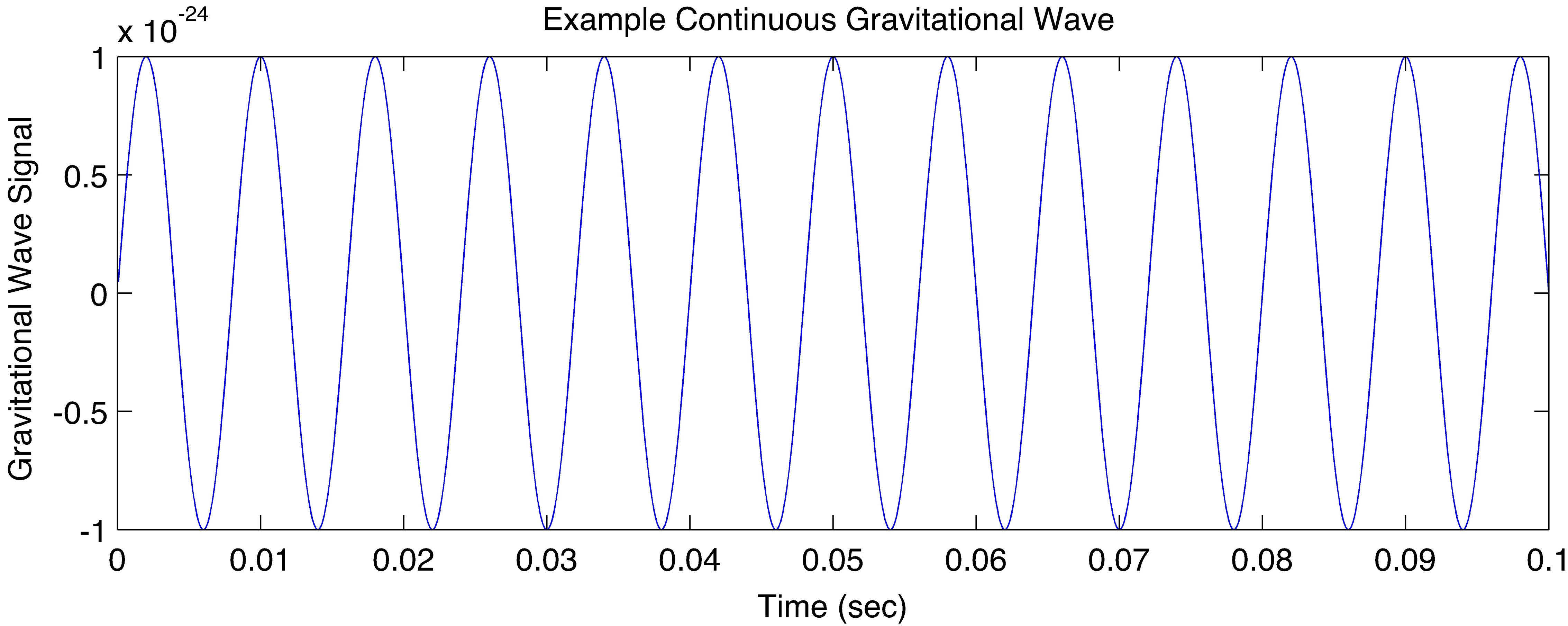
https://www.youtube.com/watch?v=QyDcTbR-kEA&t=1s

The second video is the chirp of the 2017 neutron star merger (only the last 32 seconds of the signal are included in the video).

https://www.youtube.com/watch?v=_SQbaILipjY&t=1s

Continuous gravitational waves

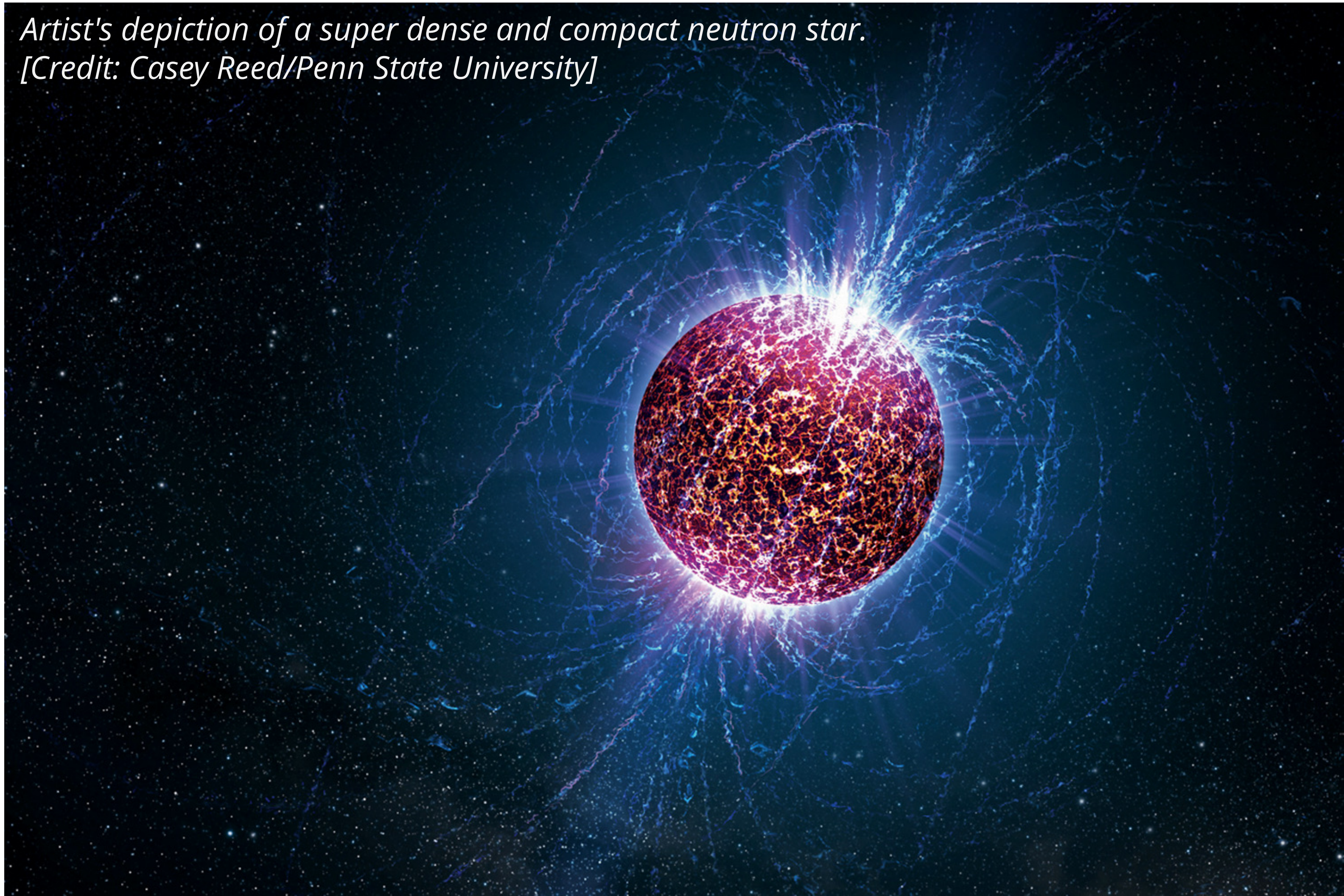
- Continuous gravitational waves are produced by systems that have a fairly constant and well-defined frequency.
- Examples of these are binary systems of stars or black holes orbiting each other (long before merger), or a single star swiftly rotating about its axis with a large mountain or other irregularity on it.
- These sources are expected to produce comparatively weak gravitational waves since they evolve over longer periods of time and are usually less catastrophic than sources producing inspiral or burst gravitational waves.
- The sound these gravitational waves would produce is a continuous tone since their frequency is nearly constant.
- In practice, the most important search targets for continuous gravitational waves are neutron stars in our own Galaxy. Once we reach sufficient detector sensitivity to find such a signal, we expect we can observe it continuously for many months or years



A short snippet of an example gravitational-wave strain signal from a continuous gravitational-wave source. Note how the peaks and troughs of the waves come at equal time intervals (hence, constant frequency) and how the height of each peak and trough is the same (constant amplitude). [Image: A. Stuver/LIGO]

NSs are extremely compact objects — with masses similar to the Sun compressed into a ball of about 10 kilometers in radius — formed in the aftermath of massive stars which undergo supernova explosions at the end of their life.

NSs may be rapid rotators, spinning with frequencies up to several hundred times per second. A rotating NS may emit detectable CWs, with a frequency of twice the star's spin frequency, if it possesses an asymmetry with respect to its rotational axis.



Artist's depiction of a super dense and compact neutron star.
[Credit: Casey Reed/Penn State University]

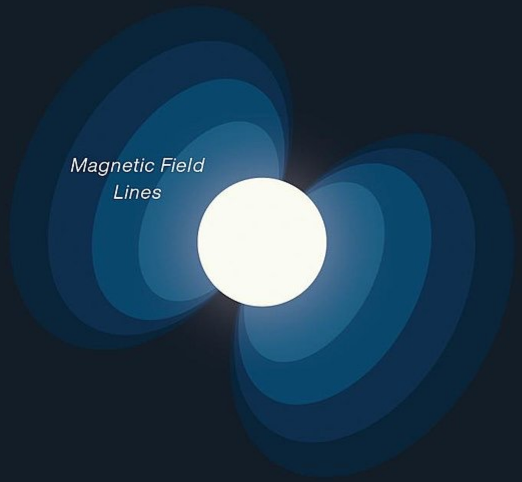
Jet Propulsion Laboratory
California Institute of Technology

DIFFERENT NEUTRON STAR TYPES

A neutron star is a dense core left behind after a massive star goes supernova and explodes. Though only about 10 to 20 miles (15 to 30 kilometers) wide, they can have three times the mass of our Sun, making them some of the densest objects in the universe, second only to black holes. A teaspoon of neutron star material would weigh 4 billion tons on Earth. There are several types of neutron stars.

MAGNETAR

A magnetar is a neutron star with a particularly strong magnetic field, about 1,000 times stronger than a normal neutron star. That's about a trillion times stronger than Earth's magnetic field and about 100 million times stronger than the most powerful magnets ever made by humans. Scientists have only discovered about 30 magnetars so far.

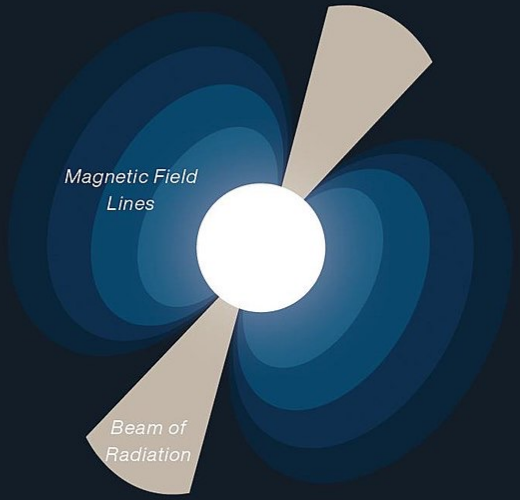


PULSAR

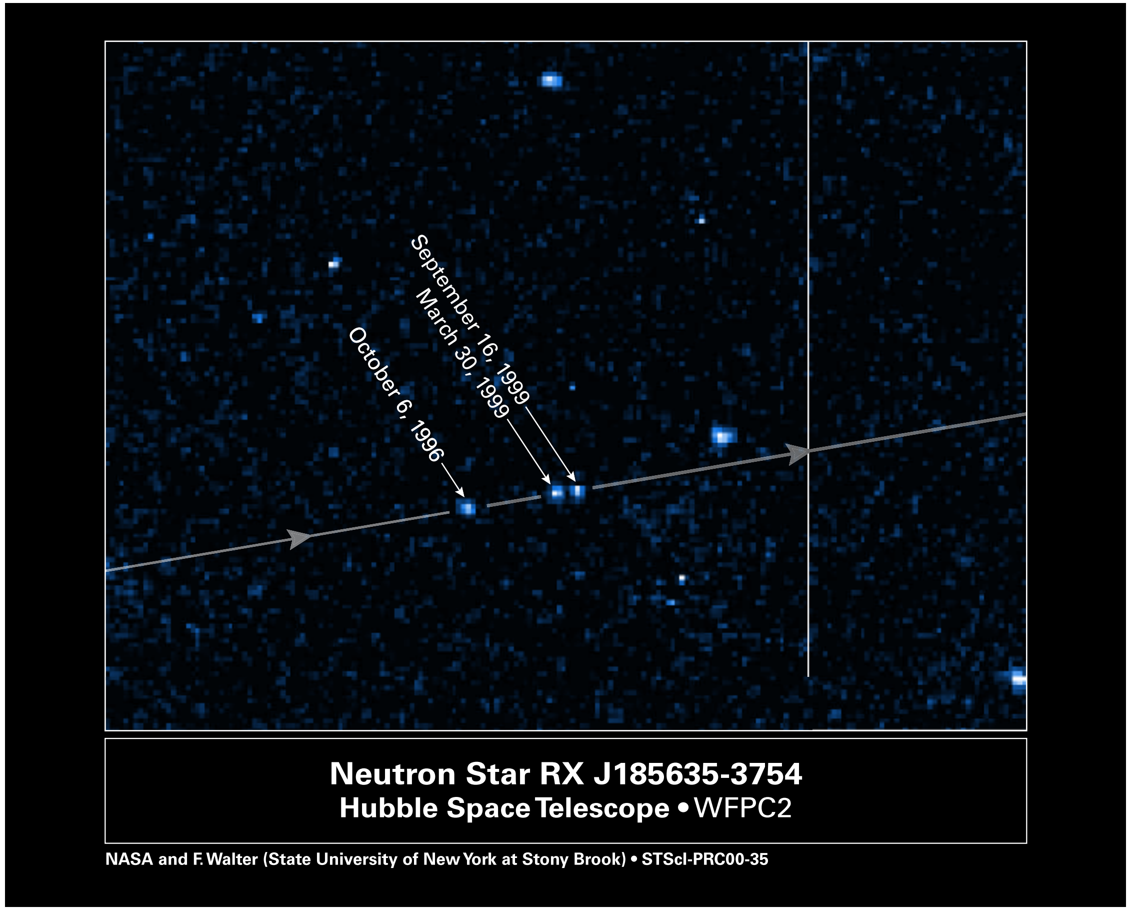
Most of the roughly 3,000 known neutron stars are pulsars, which emit twin beams of radiation from their magnetic poles. Those poles may not be precisely aligned with the neutron star's rotation axis, so as the neutron star spins, the beams sweep across the sky, like beams from a lighthouse. To observers on Earth, this can make it look as though the pulsar's light is pulsing on and off.

MAGNETAR + PULSAR

There are now six known neutron stars that are both pulsars and magnetars.

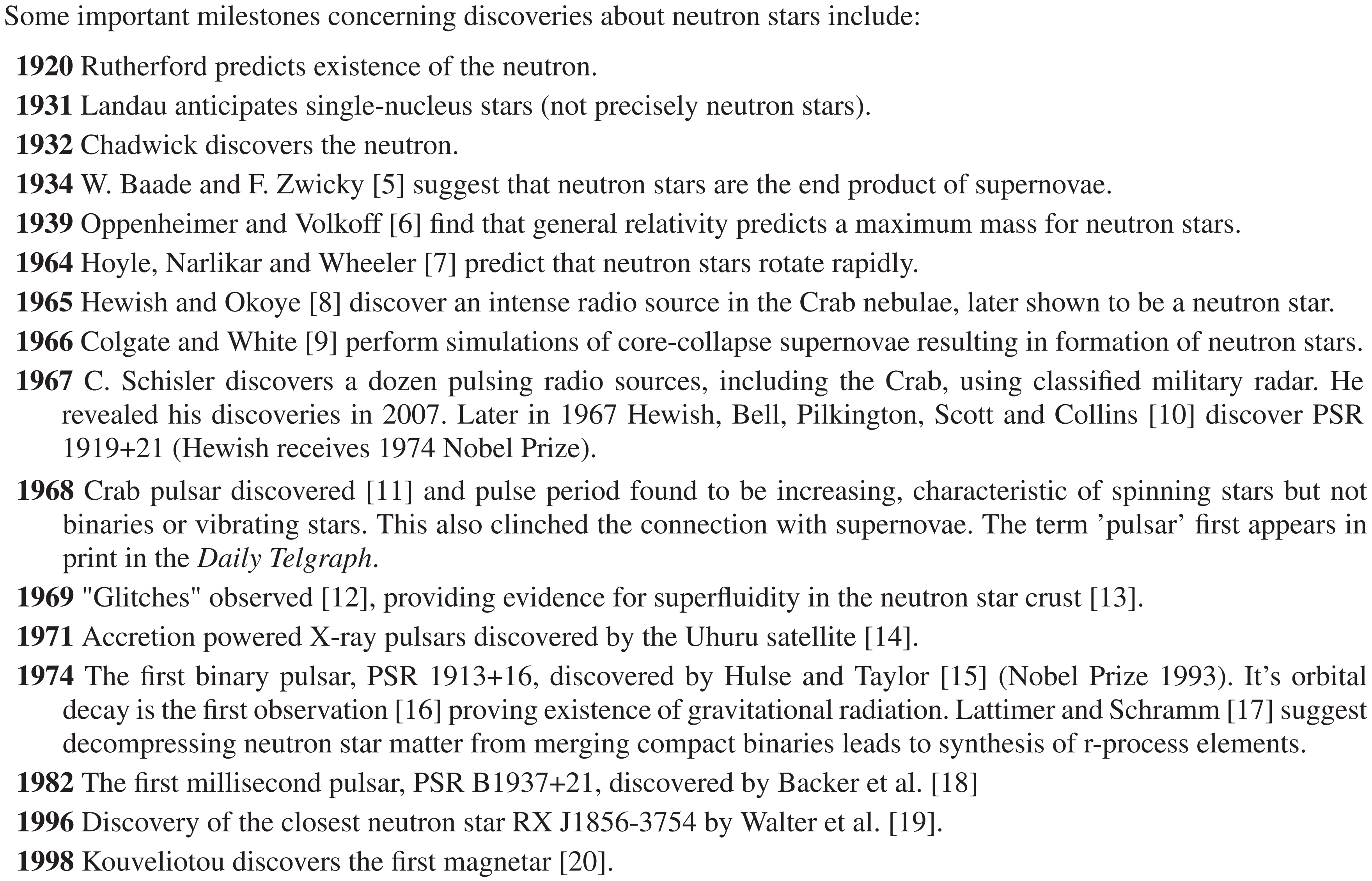


Facts accurate as of June 2020



The Motion of RX J185635-3754 – One of the nearest neutron stars to Earth

This photograph is the sum of three Hubble Space Telescope images. North is down, east is to the right. The image, taken by the Wide Field and Planetary Camera 2, is 8.8 arc seconds across (west to east), and 6.6 arc seconds top-to-bottom (south to north).
All stars line up in this composite picture, except the neutron star, which moves across the image in a direction 10 degrees south of east. The three images of the neutron star are labeled by date. The proper motion is 1/3 of an arc second per year. The small wobble caused by parallax (not visible in the image) has a size of 0.016 arc seconds, giving a distance of 200 light-years. (see also https://hubblesite.org/contents/news-releases/2000/news-2000-35.html#:~:text=Of%20the%20isolated%2C%20non%2Dpulsing,age%2C%20is%20not%20yet%20known.)



Some important milestones concerning discoveries about neutron stars include:

1920 Rutherford predicts existence of the neutron.

1931 Landau anticipates single-nucleus stars (not precisely neutron stars).

1932 Chadwick discovers the neutron.

1934 W. Baade and F. Zwicky [5] suggest that neutron stars are the end product of supernovae.

1939 Oppenheimer and Volkoff [6] find that general relativity predicts a maximum mass for neutron stars.

1964 Hoyle, Narlikar and Wheeler [7] predict that neutron stars rotate rapidly.

1965 Hewish and Okoye [8] discover an intense radio source in the Crab nebulae, later shown to be a neutron star.

1966 Colgate and White [9] perform simulations of core-collapse supernovae resulting in formation of neutron stars.

1967 C. Schisler discovers a dozen pulsing radio sources, including the Crab, using classified military radar. He revealed his discoveries in 2007. Later in 1967 Hewish, Bell, Pilkington, Scott and Collins [10] discover PSR 1919+21 (Hewish receives 1974 Nobel Prize).

1968 Crab pulsar discovered [11] and pulse period found to be increasing, characteristic of spinning stars but not binaries or vibrating stars. This also clinched the connection with supernovae. The term ’pulsar’ first appears in print in the *Daily Telgraph*.

1969 "Glitches" observed [12], providing evidence for superfluidity in the neutron star crust [13].

1971 Accretion powered X-ray pulsars discovered by the Uhuru satellite [14].

1974 The first binary pulsar, PSR 1913+16, discovered by Hulse and Taylor [15] (Nobel Prize 1993). It’s orbital decay is the first observation [16] proving existence of gravitational radiation. Lattimer and Schramm [17] suggest decompressing neutron star matter from merging compact binaries leads to synthesis of r-process elements.

1982 The first millisecond pulsar, PSR B1937+21, discovered by Backer et al. [18]

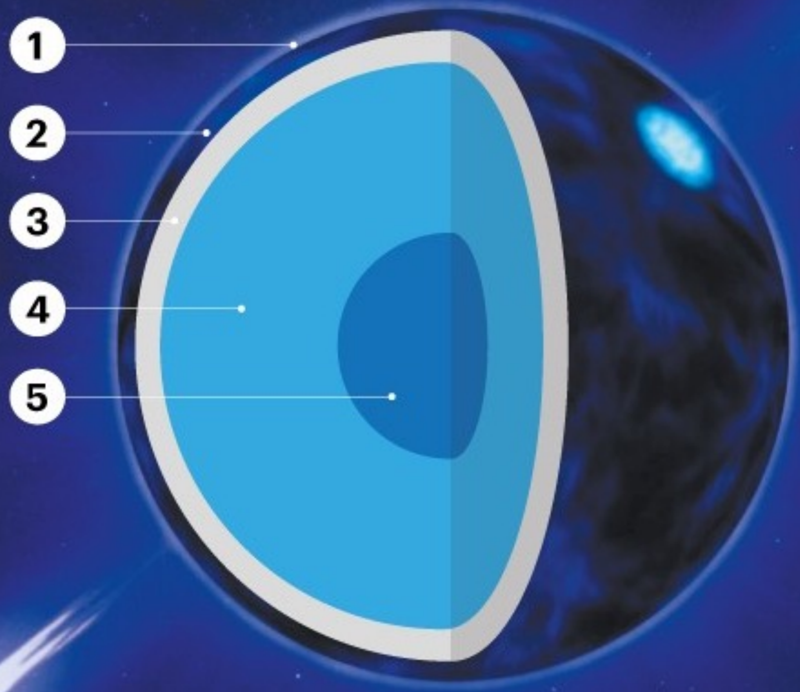
1996 Discovery of the closest neutron star RX J1856-3754 by Walter et al. [19].

1998 Kouveliotou discovers the first magnetar [20].

from J. Lattimer: "Introduction to neutron stars", AIP Conference Proceedings 1645, 61 (2015); https://doi.org/10.1063/1.4909560

DENSE MATTER

Neutron stars get denser with depth. Although researchers have a good sense of the composition of the outer layers, the ultra-dense inner core remains a mystery.



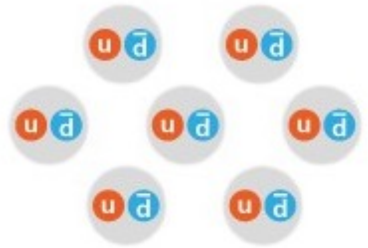
1. Atmosphere	Mostly hydrogen and helium
2. Outer crust	Atomic nuclei and free electrons
3. Inner crust	Free neutrons and electrons, heavier atomic nuclei
4. Outer core	Neutron-rich quantum liquid
5. Inner core	Unknown, ultra-dense matter

Core scenarios

A number of possibilities have been suggested for the inner core, including these three options.

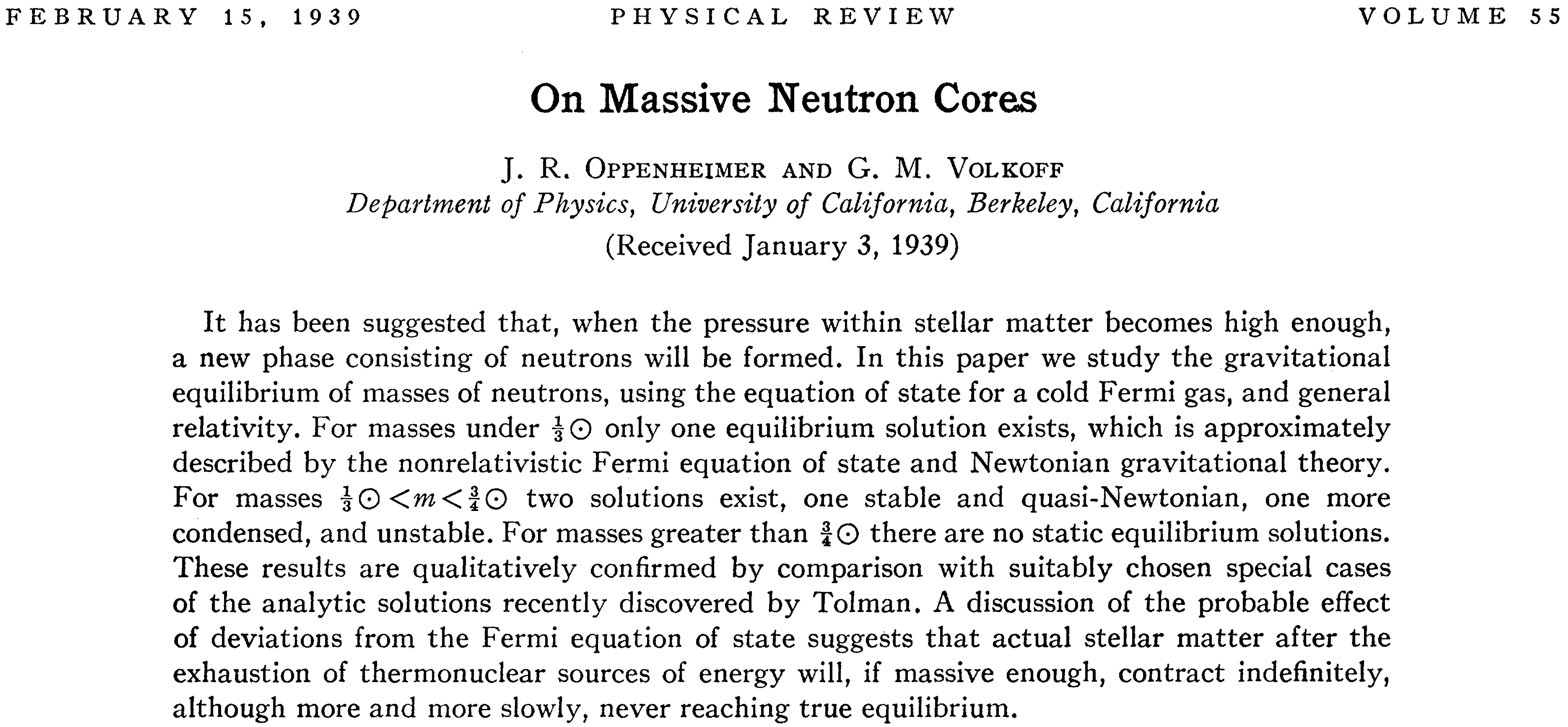
u Up quark	s Strange quark
d Down quark	d̄ Anti-down quark

Quarks
The constituents of protons and neutrons — up and down quarks — roam freely.



Bose–Einstein condensate
Particles such as pions containing an up quark and an anti-down quark combine to form a single quantum-mechanical entity.

Hyperons
Particles called hyperons form. Like protons and neutrons, they contain three quarks but include 'strange' quarks.



FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

On Massive Neutron Cores

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Department of Physics, University of California, Berkeley, California

(Received January 3, 1939)

It has been suggested that, when the pressure within stellar matter becomes high enough, a new phase consisting of neutrons will be formed. In this paper we study the gravitational equilibrium of masses of neutrons, using the equation of state for a cold Fermi gas, and general relativity. For masses under $\frac{1}{3}\odot$ only one equilibrium solution exists, which is approximately described by the nonrelativistic Fermi equation of state and Newtonian gravitational theory. For masses $\frac{1}{3}\odot < m < \frac{3}{4}\odot$ two solutions exist, one stable and quasi-Newtonian, one more condensed, and unstable. For masses greater than $\frac{3}{4}\odot$ there are no static equilibrium solutions. These results are qualitatively confirmed by comparison with suitably chosen special cases of the analytic solutions recently discovered by Tolman. A discussion of the probable effect of deviations from the Fermi equation of state suggests that actual stellar matter after the exhaustion of thermonuclear sources of energy will, if massive enough, contract indefinitely, although more and more slowly, never reaching true equilibrium.

Simple exercise: derive the Newtonian version of the Oppenheimer-Volkov equation for pressure and mass of a neutron star

pressure as a function of radial distance

$$\frac{dp}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2 r^2}$$

where we use total mass up to radius r

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(r')r'^2 dr' = \frac{4\pi}{c^2}\int_0^r \epsilon(r')r'^2 dr'$$

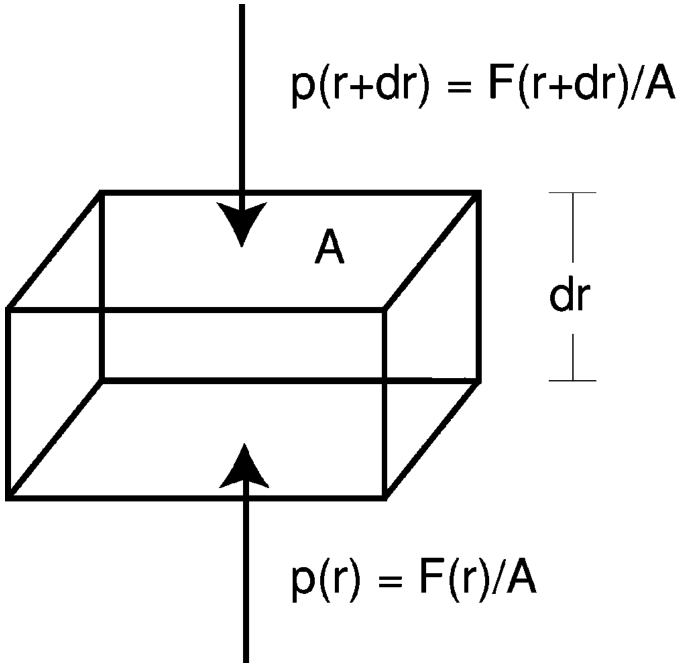
energy density

$$Ap(r) = Ap(r+dr) + \frac{G[\rho(r)Adr]\mathcal{M}(r)}{r^2}$$

⬇

$$\frac{dp}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2r^2}$$

$$\mathcal{M}(r) = 4\pi\int_0^r \rho(r')r'^2dr' = \frac{4\pi}{c^2}\int_0^r \epsilon(r')r'^2dr'$$



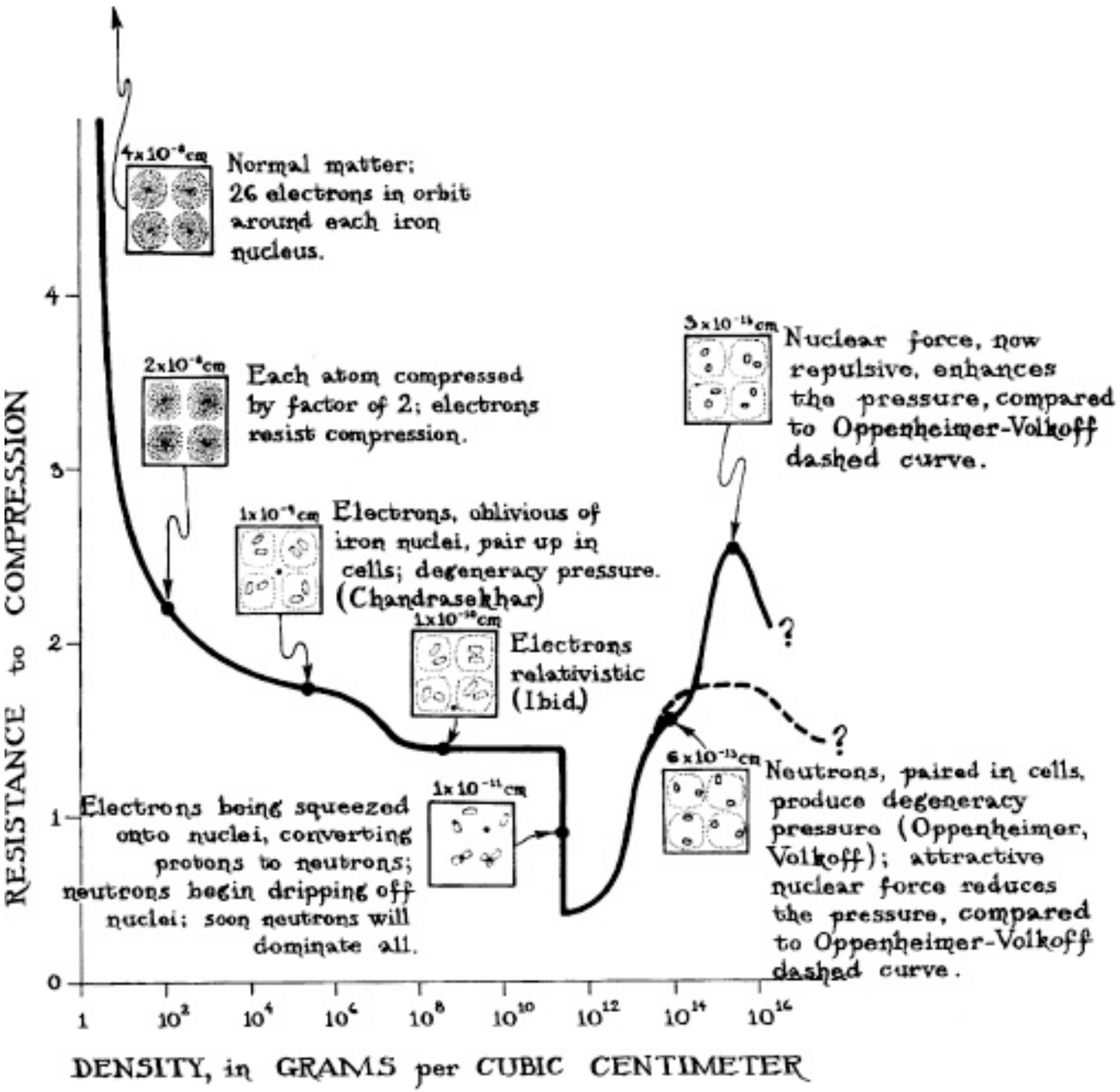
The complete, relativistic equation, contains corrections that involve the mass-energy density (the energy density and pressure are connected by the Equation Of State, EOS).

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi r^3 p/c^2)(\varepsilon+p)}{r(r-2GM/c^2)}, \qquad \frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$

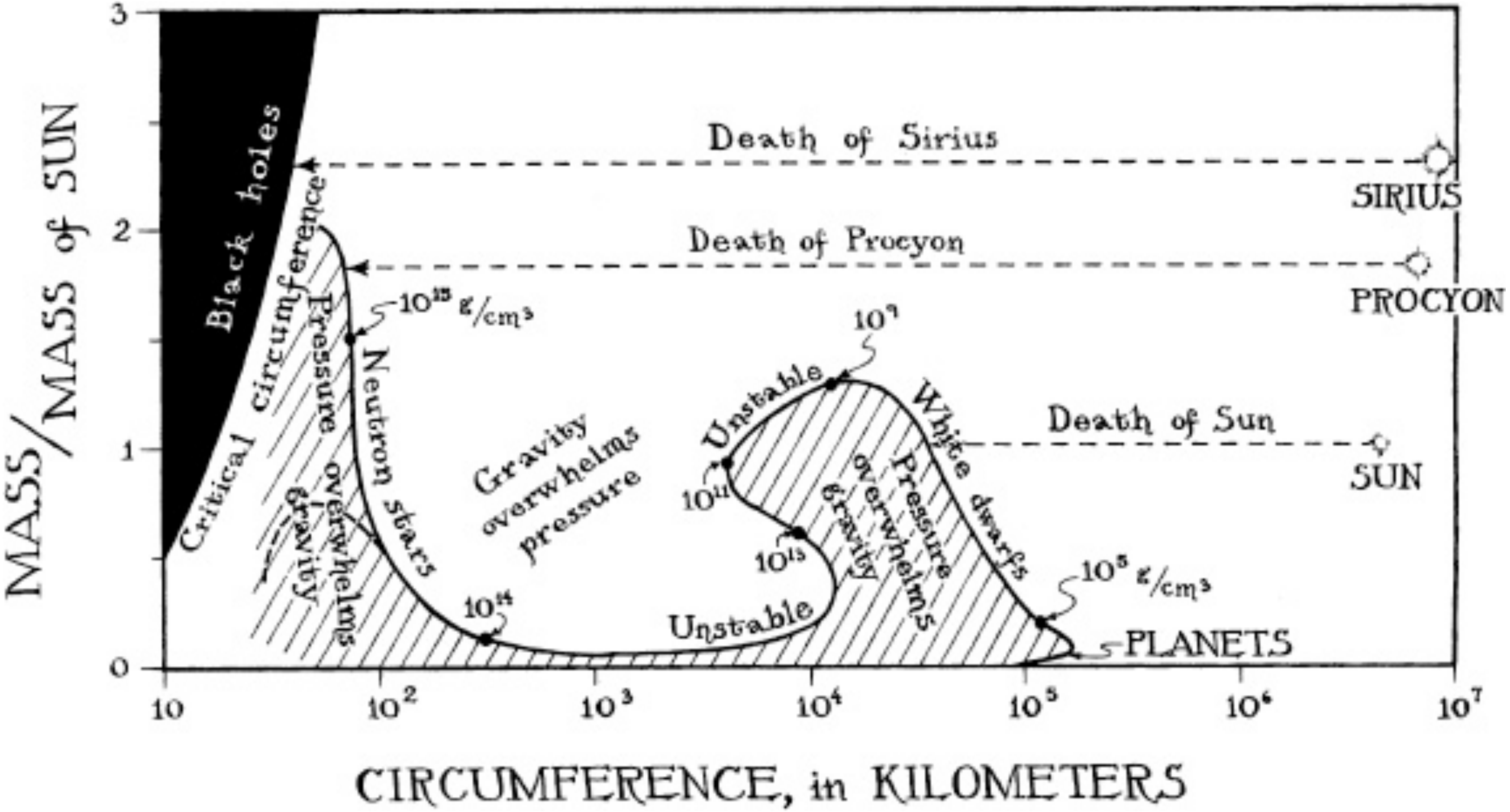
The Harrison–Wheeler Equation of State for Cold, Dead Matter

In order to study the final states of stars, we need an equation of state for the kind of matter from which dead stars are made: matter at the end point of thermonuclear evolution.

The equation of state for such matter was calculated in 1958 by B.K.Harrison and J.A. Wheeler from a knowledge of the physics of the nucleus.

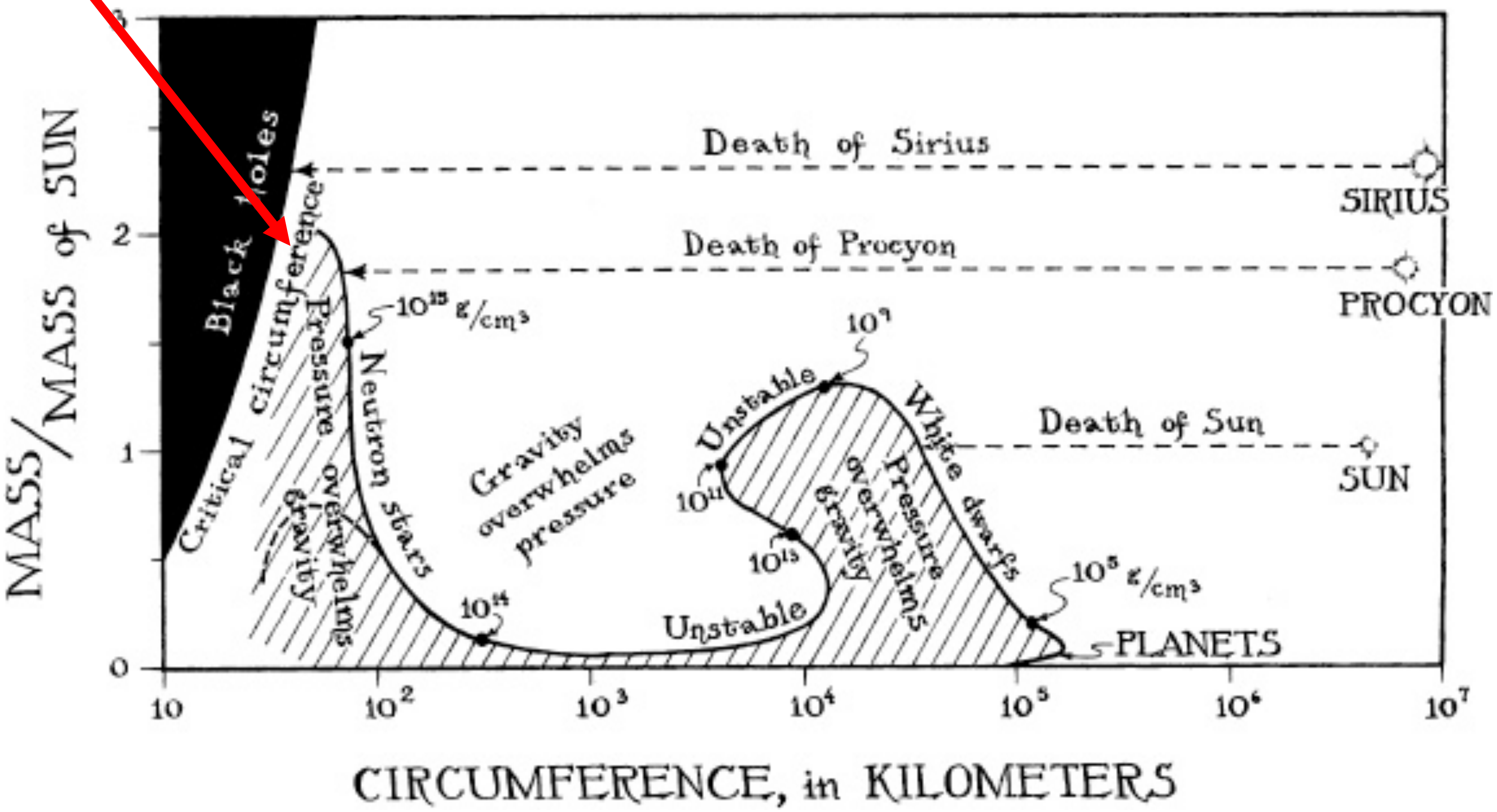


from K. S. Thorne: "Black Holes and Time Warps", Norton (1994)



from K. S. Thorne: "Black Holes and Time Warps", Norton (1994)

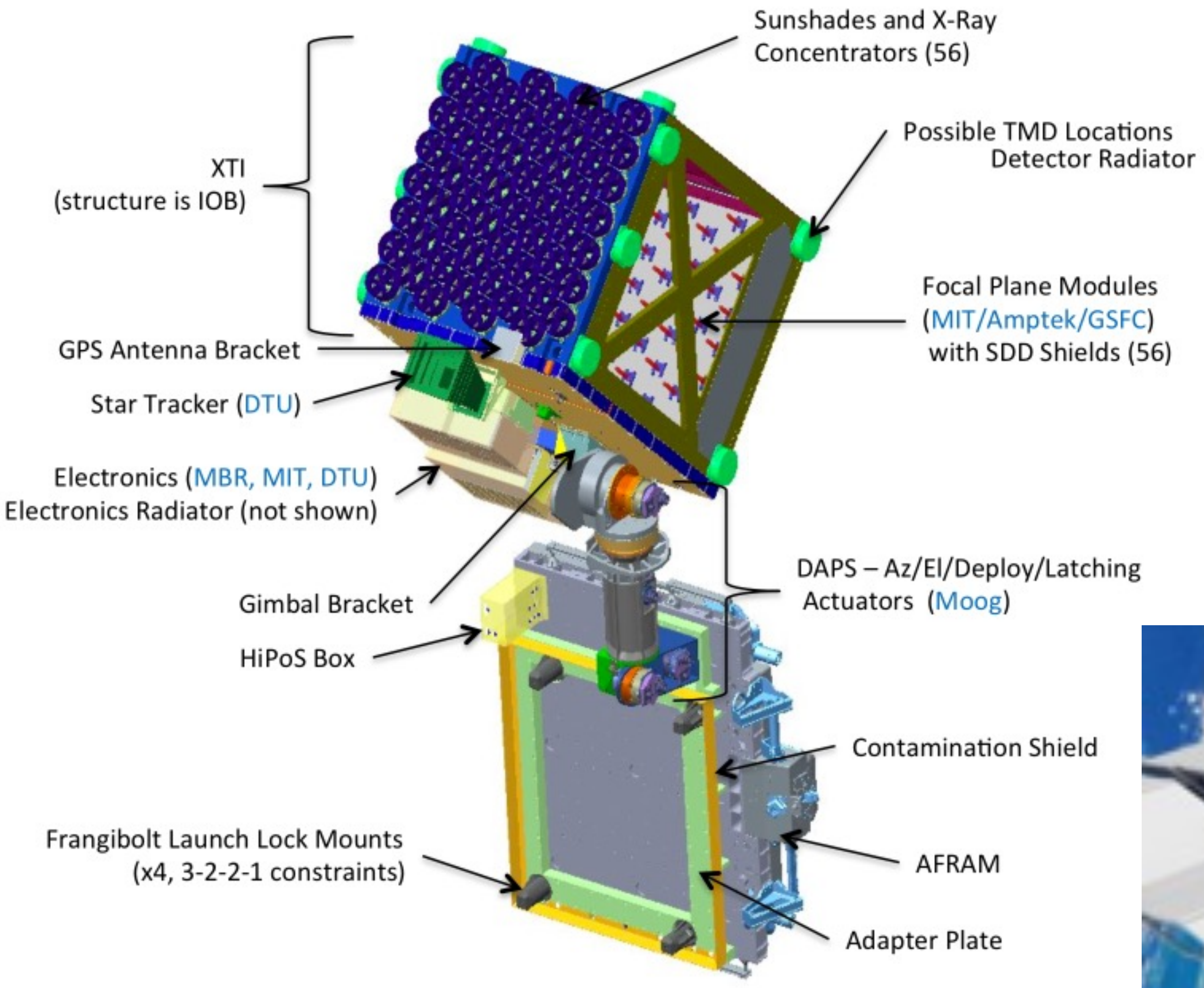
quark stars?
exotic stars?

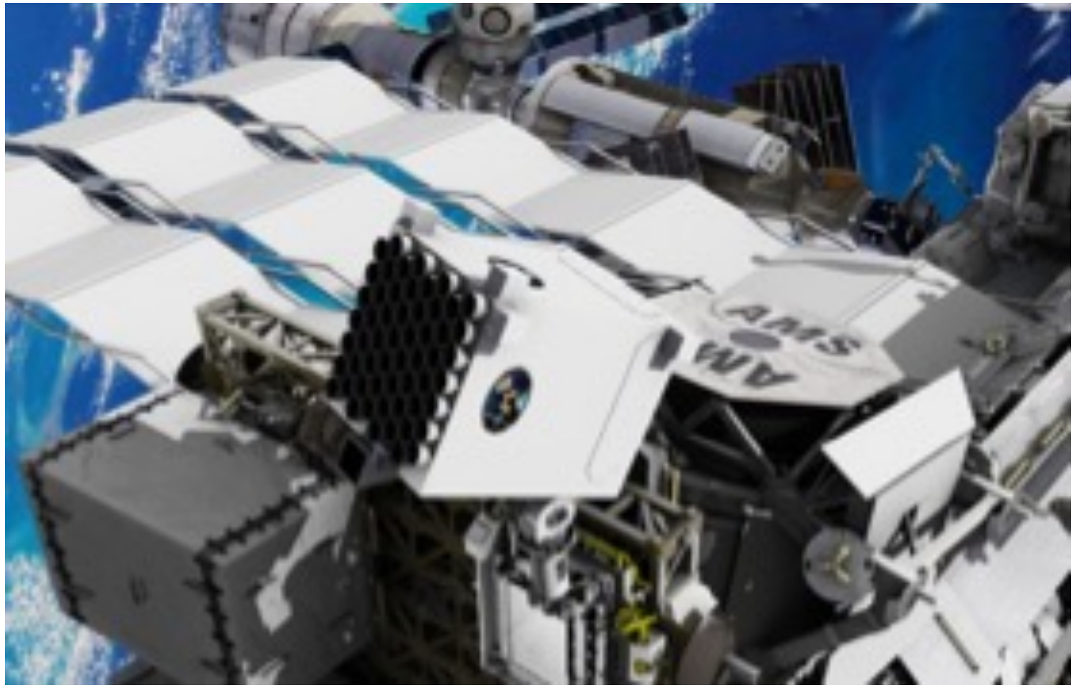


from K. S. Thorne: "Black Holes and Time Warps", Norton (1994)

The NICER instrument onboard the ISS

The X-ray Timing Instrument (XTI) consists of an array of 56 X-ray “concentrator” optics and matching silicon detectors, which record the times of arrival (100 ns resolution) and energies of individual X-ray photons (0.2-12 keV). The payload uses an on-board GPS receiver to register photon detections to precise GPS time and position, while a star-tracker camera guides the pointing system, which uses gimbaled actuators to track targets with the XTI.





Neutron star Interior Composition ExploreR
https://heasarc.gsfc.nasa.gov/docs/nicer/

J0030+0451, is an isolated pulsar that spins roughly 200 times per second and is 337 parsecs (1,100 light years) from Earth, in the constellation Pisces. M ≈ 1.3 – 1.4 Mo; radius ≈ 13 km

Hotspots rotate in two scenarios for the pulsar J0030+0451, based on analysis of NICER data.Credit: NASA’s Goddard Space Flight Center/CI Lab

Individual neutron stars, whether isolated or in binary systems, can radiate gravitational waves if they spin (which almost all of them do) *and* if they are somehow significantly non-axisymmetric.

The non-axisymmetries:

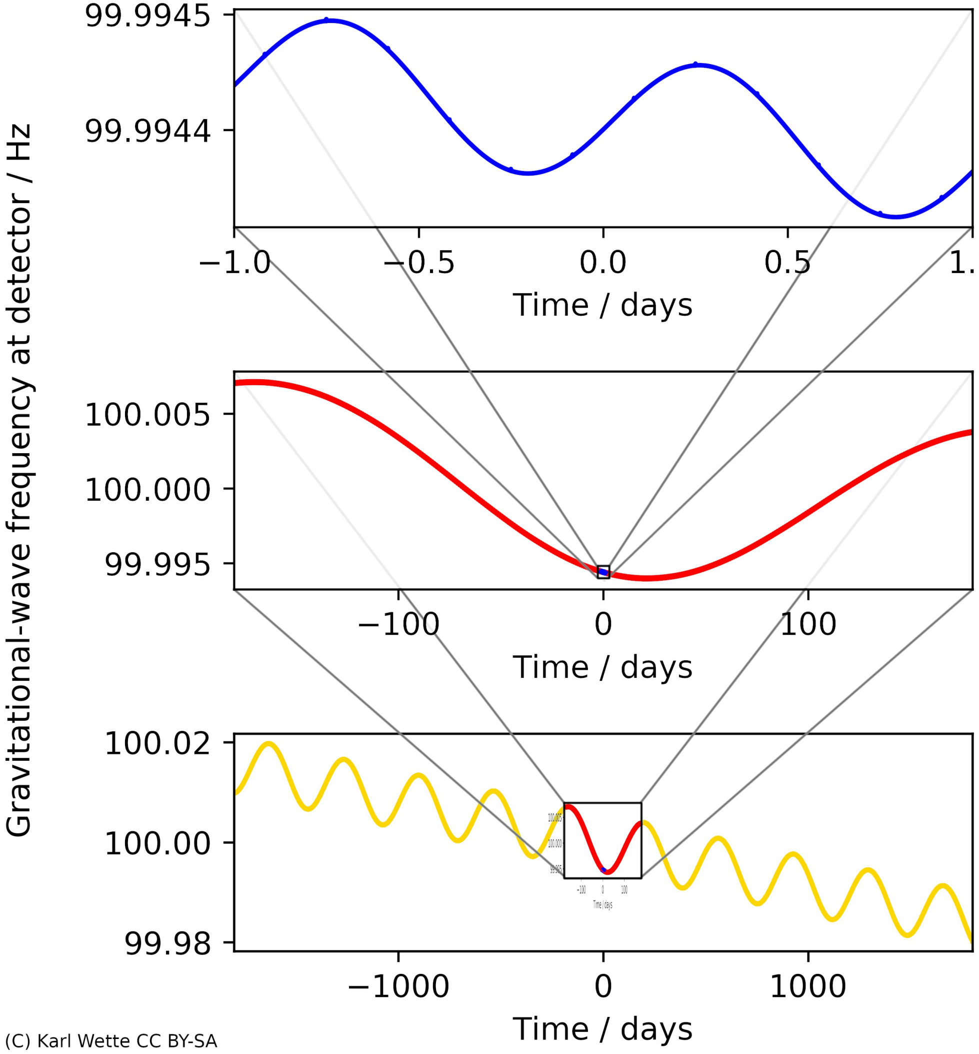
- may come from **irregularities in the crust**, perhaps from strains that have built up as the stars have spun down or perhaps from irregularities associated with their formation that became frozen in as the star cooled;
- they can be **dynamical**, such as normal modes of pulsation that are excited in some way, or precession that is driven by the accretion of angular momentum;
- they may be **associated with the off-axis magnetic field** of an active pulsar.

Over longer durations, the frequency of the signal will slowly change, for two reasons.

- The first reason is that, as the neutron star emits gravitational and electromagnetic waves, it loses energy which causes it to rotate more slowly.
- The second reason is that the detector here on Earth is moving with respect to the neutron star, which changes the frequency of the gravitational waves observed in the detector.

The figure shows the long-term frequency evolution of a continuous gravitational-wave signal. The **top panel** shows the frequency, in blue, changing with a **daily** cycle due to the rotation of the Earth. The **middle panel** zooms out to show the frequency, in red, changing on a **year's time** scale due to the orbit of the Earth around the Sun. The **bottom panel** zooms out further to show how the frequency, in green, slowly decreases due to the rotation of the neutron star itself slowing down **over many years**.

Tracking all possible frequency changes is what makes the detection of continuous gravitational waves a computational challenge. [Image: K. Wette]



- According to current theoretical models and previous LIGO–Virgo CW searches, such an asymmetric bulge (sometimes also called a “mountain”, “raised plateau” or deformation on one side of the NS) would be at most just a few centimeters high.
- The bigger the NS asymmetry is, the stronger GWs it generates. For example, a bulge of a few centimeters corresponds to a deformation of a NS from spherical symmetry — also called the ellipticity — of a few parts per million.
- Determining a NS ellipticity is not easy, but once detected it will provide a truly unique insight into the properties of the extremely dense matter and strong magnetic fields of which the star is composed. Currently these properties are very poorly known.
- In addition to about **3000 NSs known** to astronomers because they are electromagnetically bright (for example, as radio pulsars), our Galaxy, the Milky Way, is estimated to contain as many as **100 million NSs not visible** in electromagnetic radiation because they are either too faint to be detected, or their electromagnetic emission is not directed towards Earth. However, if any of these NSs are sufficiently asymmetric, they may emit GWs detectable by our interferometers.

The observed spindown of radio pulsars is presumed to be driven primarily by the emission of a wind of energetic particles and of low-frequency electromagnetic waves from the spinning dipole magnetic moment.

The radio waves that we observe carry very little energy, so we have no direct observations of the spindown mechanism. This leaves room for the possibility that gravitational radiation contributes a significant amount to the spindown as well.

By using the spindown to place an upper limit on *h*, essentially from equation

$$h = \left(\frac{G}{\pi^2 c^3}\right)^{1/2} E^{1/2} \tau^{-1/2} f^{-1} r^{-1}$$

where the energy is the kinetic energy of rotation of the pulsar and the timescale τ is the spindown timescale, one finds upper limits on the gravitational radiation from the Crab and Vela pulsars that is of the order of 10^{-24}.

The energy loss rate is also related to the non-axisymmetric ellipticity $\epsilon = 1 - a_2/a_1$, where a_1 is the semimajor axis of the equatorial section and a_2 the semiminor axis. The formula shows the dependence of the ellipticity on the spin period *P* of the pulsar and on its rate of change by

$$\epsilon = 0.0057 \left[\frac{P}{1\ \mathrm{s}}\right]^{3/2} \left[\frac{\dot{P}}{10^{-15}}\right]^{1/2}$$

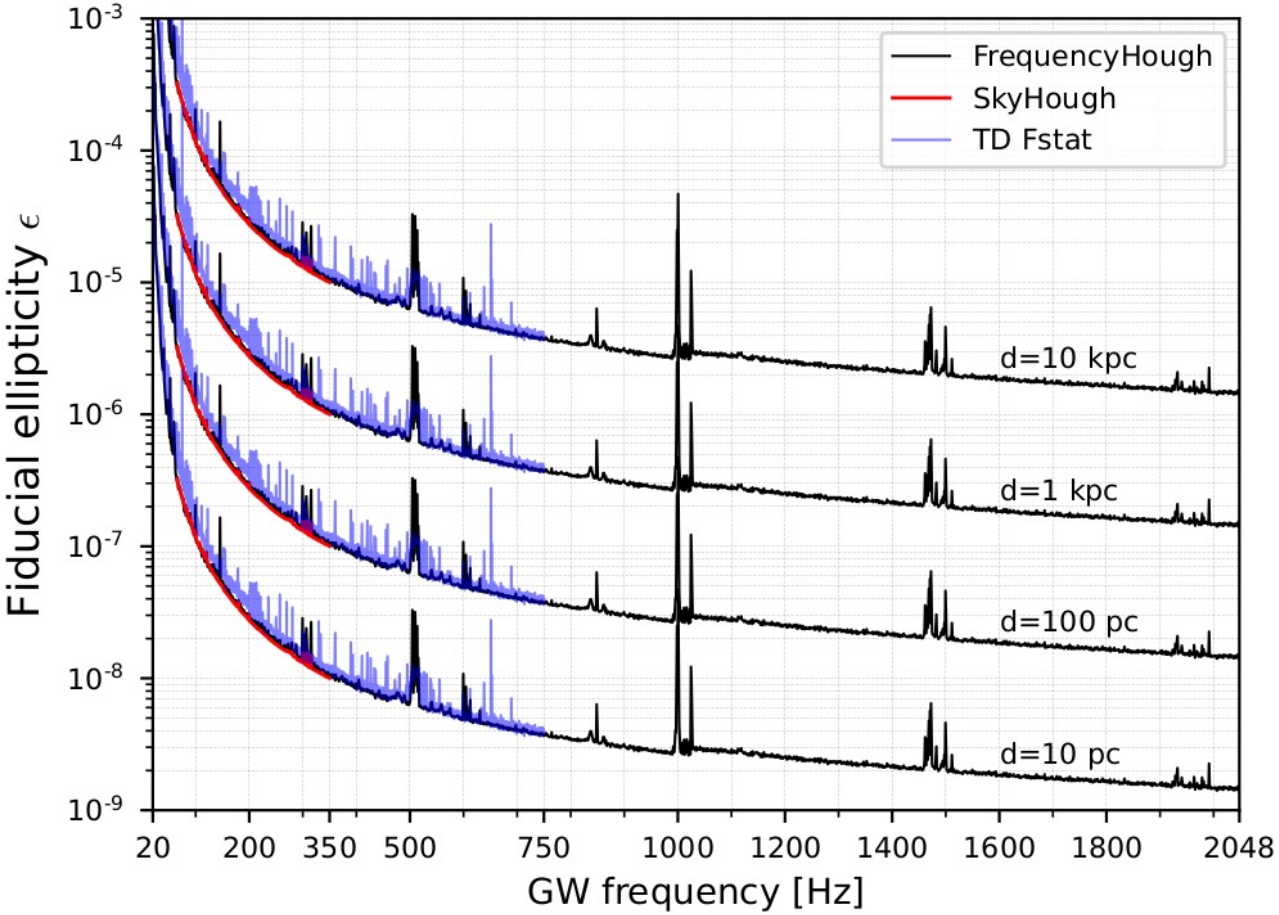
The figure shows additional astrophysical information from the search performed during O3. It displays the relation between the frequency, spin-down and distance of a potentially detectable source. For instance, at frequency 200 Hz we would be able to detect a CW signal from a NS within a distance of 100 pc (parsecs) if its ellipticity were at least 3×10^{-7}.

Similarly, in the middle frequency range, around 550 Hz, we would be able to detect the CW signal up to a distance of 1 kpc (1000 parsecs), with ellipticity greater than 5×10^{-7}.

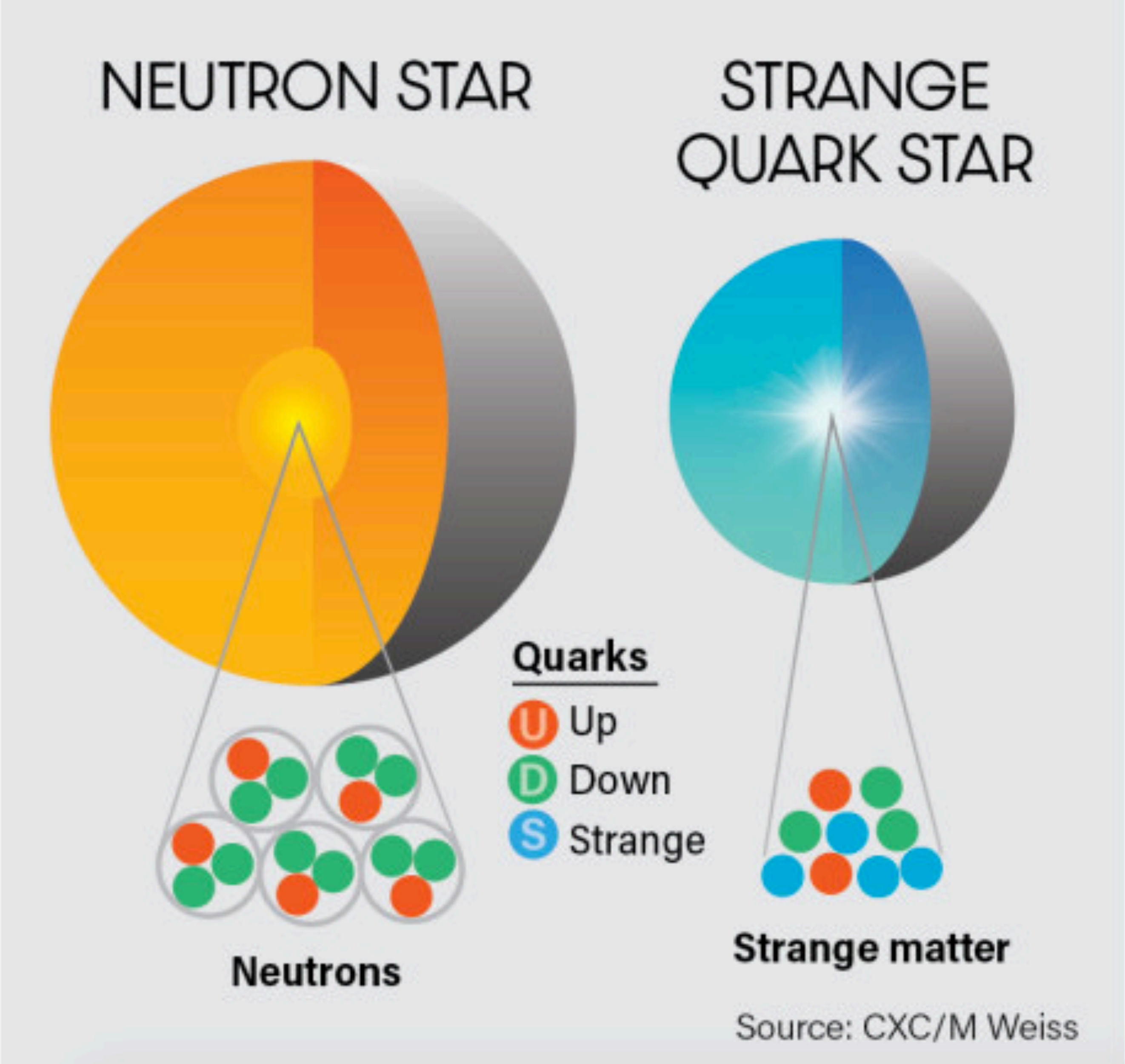
For comparison, the radius of our Galaxy is about 15 kpc.

Although no detection can be claimed, our results are nevertheless interesting from the astrophysical point of view and shed more light on the properties of Galactic NSs.

Our upper limits are starting to probe the range of ellipticities up to $10^{-7} - 10^{-6}$ for some models of younger NSs in which the deformation is not supported by the elasticity of the crust, but by a non-axisymmetric magnetic field.

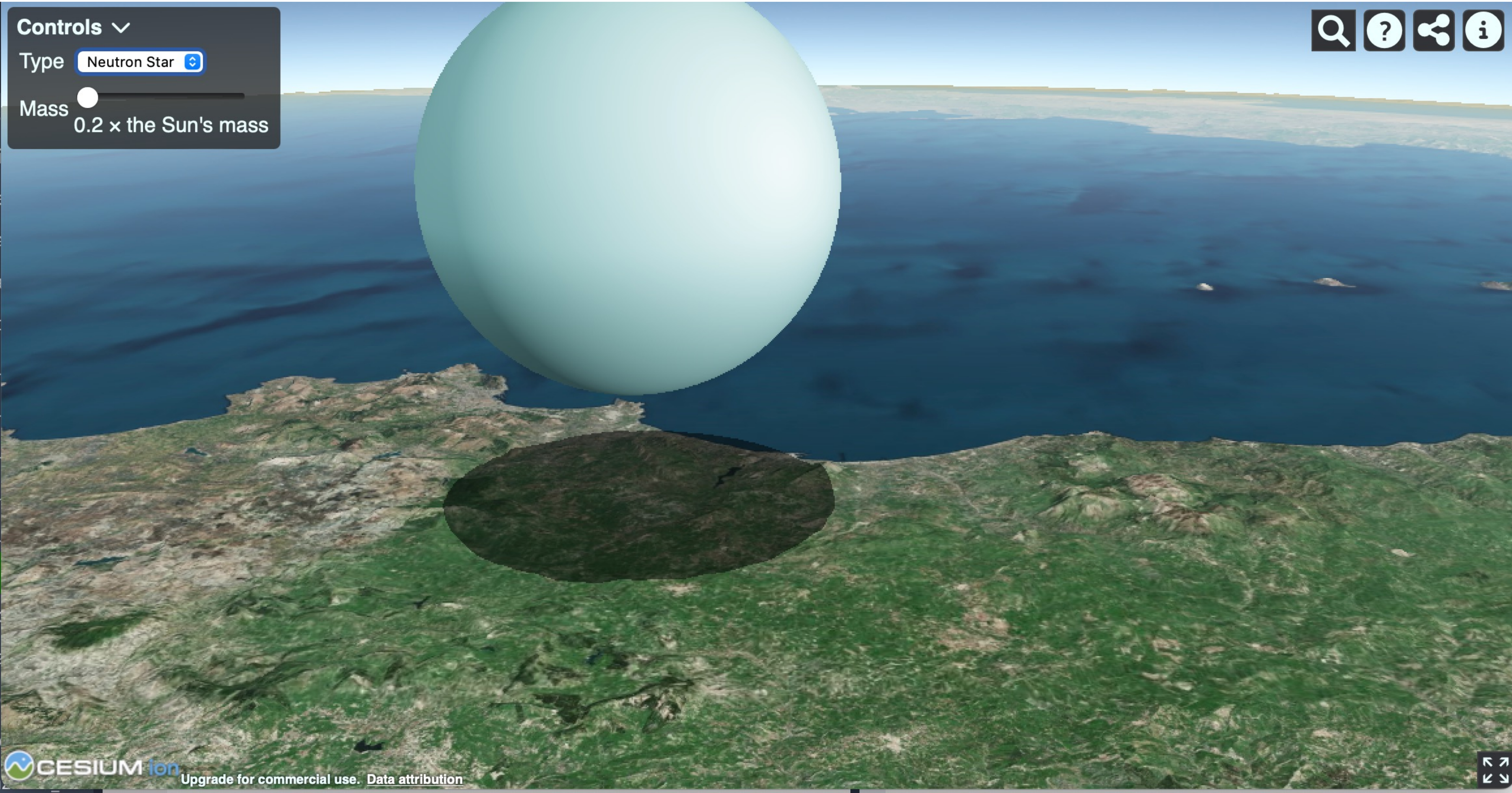


Detectable neutron star ellipticity during the run O3 of the LVK Collaboration, as a function of the GW frequency at distances of 10 kpc, 1 kpc, 100 pc, and 10 pc (from top to bottom). Results from the FrequencyHough pipeline are marked in black, from SkyHough in red and from the Time-Domain F-statistic in blue.



Source: CXC/M Weiss

A neutron star in my city https://ns-in-my-city.daniel-wysocki.info



Type: Neutron Star

Mass: 0.2 × the Sun's mass

Burst gravitational waves

There are several conjectured sources of *burst signals*, here we concentrate on supernovae only.

Core-Collapse Supernovae (CCSNe) are spectacular deaths of massive stars with masses larger than 8 times the mass of our Sun (or, 8 solar masses).

- These stars burn hydrogen in their cores over millions or billions of years, in the process creating heavier elements up to iron.
- The iron builds up until it creates a so-called iron core in its center.
- When such an iron core is around 1.5 solar masses, its gravity becomes so strong that it exceeds the electron pressure of atoms and the star's core collapses under its own weight.
- Under that tremendous pressure, the electrons penetrate the iron atoms, interacting with protons to create neutrons and neutrinos.
- The neutrons stay in the star's core, but the very light neutrinos leave the collapsed core en masse.
- This massive flux of neutrinos is believed to drive the inevitable explosion of the star by heating it from the inside.

SNs are dim GW sources; they are also quite rare

People have been observing supernovae for millennia, but the main mechanism behind these powerful explosions is not yet fully understood.

Theorists model supernovae and calculate what gravitational-wave signals, or waveforms, from these events would look like.

Simulations of core-collapse supernovae show that GWs between (10^2– 10^3) Hz can be produced. Dimmelmeier et al calculate the average maximum amplitude of GWs for a supernova at distance r as

$$h_{\max} = 8.9 \times 10^{-21}\left(\frac{10\ \mathrm{kpc}}{r}\right)$$

(diameter of the Milky Way, roughly 27 kpc).

The event rate for SNs is approximately 5×10^{-4} Mpc^{-3} yr^{-1}.

https://www.youtube.com/watch?v=XzaYNOKK6Xk

Onset of neutrino-driven convection in a 15 solar mass core-collapse supernova (CCSN) simulation done with the CHIMERA code.

https://www.youtube.com/watch?v=dfROBwCDKtM https://d2r55xnwy6nx47.cloudfront.net/uploads/2021/01/Supermodels-Inside.mp4

Swirling matter surrounds the core of a supernova in the first half second after core collapse. In this simulation, the matter is colored by entropy, a measure of disorder. (Hotter colors like red indicate higher entropies.) Because of the turbulence, the explosion isn’t symmetric.

Credits D. Vartanyan, A. Burrows. Thanks to ALCF, D. Radice and H. Nakagura

Stochastic background

Stochastic gravitational waves are the relic gravitational waves from the early evolution of the universe. Much like the Cosmic Micro-wave Background (CMB), which is likely to be the leftover light from the Big Bang, these gravitational waves arise from a large number of random, independent events combining to create a cosmic gravitational wave background.

The Big Bang is expected to be a prime candidate for the production of the many random processes needed to generate stochastic gravitational waves (and the CMB), and therefore may carry information about the origin and history of the universe.

If these gravitational waves truly originated in the Big Bang, these waves will have been stretched as the universe expanded and they can tell us about the very beginning of the universe—they would have been produced between approximately 10^{-36} to 10^{-32} seconds after the Big Bang, whereas the CMB was produced approximately 300,000 years after the Big Bang. The sound these gravitational waves would produce is a continuous noise (much like static) and will be same from every part of the sky (just like the CMB).

Similar backgrounds could be produced by a combination of many simultaneous inspirals, bursts, or continuous signals from throughout the Universe.

The noise PSD of the stochastic gravitational wave background (SGWB) is

fraction of the closure energy density in the form of GWs

$$S_{\mathrm{gw}}(f) = \frac{3H_0^2}{10\pi^2} f^{-3}\Omega_{\mathrm{gw}}(f)$$

A background of gravitational waves appears in a single detector as simply another source of noise. There are two ways to identify it:

- In a single detector, if one has confidence in the characterization of the instrumental and environmental noise, and if the observed noise is larger, then one could attribute the excess noise to a background of gravitational waves. For ground-based detectors, the expected gravitational-wave noise level is so low that it will not be seen against expected instrumental noise sources.
- Using two detectors, one can cross-correlate their output data streams, essentially by multiplying them together and integrating over an observation time *T* .
  The instrumental noise, assumed independent, largely cancels out, while the gravitational-wave noise, being the same in both detectors, sums systematically.
  This technique works, provided the two detectors are close enough together to respond in the same way to any given component of the stochastic gravitational-wave field.
  In practice, detectors will be separated by significant distances, and this causes their mutual gravitational-wave response to decorrelate somewhat.